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TABLE OF CONTENTS

Probability	47	Classical Thermodynamics, Heat Transfer	92
Statistics	50	Quantum Mechanics	93
		Relativity	94
Numerical Methods	56	Astronomy	97
Computing Machines	59	Geophysics	97
Mechanics of Particles and Systems	59	Operations Research, Econometrics, Games	98
Statistical Thermodynamics and Mechanics	61	Biology and Sociology	100
Elasticity, Plasticity	63	Information and Communication Theory	100
Structure of Matter	69	Servomechanisms and Control	102
Fluid Mechanics, Acoustics	70		
Optics, Electromagnetic Theory, Circuits	77	Author Index	105

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References to reviews in Mathematical Reviews before volume 20 (1959) are by volume and page number, as MR **19**, 532: from volume 20 on, by volume and review number, as MR **20** #4387. Reviews reprinted from Applied Mechanics Reviews, Referativnyĭ Žurnal, or Zentralblatt für Mathematik are identified in parentheses following the reviewer's name by AMR, RŽMat (or RŽMeh, RŽAstr. Geod.), Zbl, respectively.

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61
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Mathematical Reviews

Vol. 22, No. 1B January, 1961 Reviews 240–658

PROBABILITY

See also A167, A202, 303, 478, 511, 642, 649, 657a–b.

240:

Gnédenko, B. V.; Khintchine, A. Ia. ★Introduction à la théorie des probabilités. Traduit par Marc Gilliard. Monographies Dunod, no. 26. Dunod, Paris, 1960. viii+157 pp. 12.50 NF.

Translated, with new examples added by the first author, from *Elementarnoe vvedenie v teoriyu veroyatnostei* [4th ed., GITTL, Moscow, 1957; MR **19**, 889].

241:

Haight, Frank A. The generalized Poisson distribution. Ann. Inst. Statist. Math. Tokyo **11** (1959), 101–105.

The generalized Poisson distribution is defined here to be the distribution of the largest integer less than or equal to X/k, where X has a Poisson distribution with parameter λ, and where k is a positive integer. Calculation of the first moment is discussed with a view of providing an estimate of λ when k is known.

T. S. Ferguson (Princeton, N.J.)

242:

Ruben, Harold. A theorem on the cumulative product of independent random variables. Proc. Cambridge Philos. Soc. **55** (1959), 333–337.

The author proves the following result. Let $u_1, u_2, \cdots$ be a sequence of independent random variables with $E(u_i)=1$. Let $S_1, S_2, \cdots$ be a sequence of mutually exclusive events such that S_k depends only on $u_1, u_2, \cdots, u_k$ with $\sum_1^\infty P(S_i)=1$. Define the random variable $n=n(u_1, u_2, \cdots)$ on the infinite-dimensional space of the u_i to equal k when S_k occurs. Define further $U_k=\prod_1^k u_i$, $P_N=P(n\leqq N)$, and assume that $E(U_n)$ exists. Then

$$E(U_n) = 1 \tag{1}$$

if, and only if,

$$\lim_{N\to\infty} (1-P_N)E(U_N \mid n > N) = \lim_{N\to\infty} \int_{R_N} u_1 u_2 \cdots u_N dP = 0, \tag{2}$$

where R_N is the point set in N-space (u_i real-valued) or in $2N$-space (u_i complex-valued) defined by $n>N$.

J. Wolfowitz (Ithaca, N.Y.)

243:

Johnson, N. L. A proof of Wald's theorem on cumulative sums. Ann. Math. Statist. **30** (1959), 1245–1247.

Let $\{z_i\}$ be a sequence of independent random variables with common expected value μ and common finite variance σ^2. Let n be a positive integer-valued random variable such that the event $\{n>i\}$ is determined by conditions on $z_1, \cdots, z_i$. The main part of the paper presents several additional assumptions, each, together with those already mentioned, assuring the validity of the equation

$$E(\sum_{i=1}^{n} z_i - n\mu)^2 = \sigma^2 E(n). \tag{*}$$

One such assumption is that $E(n^r)$ be finite for some $r>2$. Equation (*) has been proved by Wald [same Ann. **17** (1946), 493–497; MR **8**, 284], Wolfowitz [same Ann. **18** (1947), 215–230; MR **9**, 49], and others, under various sets of conditions. {Two errors were found. It is clear that the author intended to assume that $E(n)$ be finite in Theorem 1. Also, in condition (vi)", P_i should be replaced by $\sum_{j=1}^{\infty} P_j$.}

D. L. Burkholder (Urbana, Ill.)

244:

Okamoto, Masashi. A convergence theorem for discrete probability distributions. Ann. Inst. Statist. Math. Tokyo **11** (1959), 107–112.

A modification of Scheffé's "useful" convergence theorem, which will apply directly to a sequence of integer valued random variables converging to an absolutely continuous distribution, is proved and extended to lattice valued random vectors in finite dimensional Euclidean space. As examples, the asymptotic normality of the hypergeometric distribution and of the conditional distribution, given the marginals, of some contingency tables is demonstrated.

T. S. Ferguson (Princeton, N.J.)

245:

Geffroy, Jean. Contribution à la théorie des valeurs extrêmes. II. Publ. Inst. Statist. Univ. Paris **8** (1959), 3–65.

This paper is the second of two parts in which the author gives a detailed presentation of previously announced results on extreme values. References to his earlier work are listed in the review of the first part [same Publ. **7** (1958), no. 3/4, 37–121; MR **21** #3911]. The chapter headings of the completed paper are: Upper and lower asymptotic bounds of extreme values; A study of stability in probability of extreme values; Almost complete stability of extreme values; Almost sure stability of extreme values; Some considerations of asymptotic independence of two random variables and their application to the study of the stability of the sample median and range; A study of certain limit laws; Properties of two-dimensional samples. Chapters I through IV comprise the first of the two papers.

R. Pyke (New York)

246:

Cramér, Harald. ★Aus der neueren mathematischen Wahrscheinlichkeitslehre. Arbeitsgemeinschaft für Forschung des Landes Nordrhein-Westfalen, Heft 76a. Westdeutscher Verlag, Cologne and Opladen, 1959. 28 pp. DM 2.60.

Dieser Vortrag soll nichts Neues bieten, stellt jedoch in leicht fasslicher Form die Grundbegriffe der modernen Wahrscheinlichkeitstheorie dar und erläutert dann in klarer Weise an drei Beispielen das Wesen eines stochastischen Prozesses: am Wiener Prozess, dem Poissonschen Prozess und den stationären Prozessen.

L. Schmetterer (Hamburg)

247:

Grenander, Ulf. Some non linear problems in probability theory. Probability and statistics: The Harald Cramér volume (edited by Ulf Grenander), pp. 108–129. Almqvist & Wiksell, Stockholm; John Wiley & Sons, New York; 1959. 434 pp. $12.50.

The author discusses in heuristic terms a number of nonlinear problems in probability theory and techniques available for solving them. The first problem is that of determining the spectral characteristics of an instantaneous nonlinear function $y(t)=H(x(t))$ of a stationary process $x(t)$. A few remarks are also made about non-instantaneous nonlinear filters. Nonlinear stochastic differential equations and differential equations with random coefficients are then considered. There are sections on the problems of determining limit laws for products of random matrices and the spectrum of random operators.

M. Rosenblatt (Providence, R.I.)

248:

Takács, L. On a sojourn time problem in the theory of stochastic processes. Trans. Amer. Math. Soc. **93** (1959), 531–540.

Let the abstract state space of a stochastic process $x(t)$, $t\geqq 0$, consist of two disjoint sets A, B, and let ξ_n, η_n be the respective occupation times of the nth sojourn of the $x(t)$ in A and B. Assume the ξ_n and η_n are independent, and assume further there exist constants $A_1\geqq 0$, $A_2>0$, $a>0$ such that $(\xi_1+\xi_2+\cdots+\xi_n)/A_2n^a$ has a non-degenerate limiting distribution, and similarly for the sequence η_n with constants $B_1\geqq 0$, $B_2>0$, $b>0$. Then there exist constants M_1, M_2, m such that if $\beta(t)$ is the occupation time of $x(\tau)$ in the set B, $0\leqq\tau\leqq t$, then $(\beta(t)-M_1t)/M_2t^m$ has a non-degenerate limiting distribution. The author gives the twelve possible forms it may have.

D. A. Darling (Ann Arbor, Mich.)

249:

Gabriel, K. R. The distribution of the number of successes in a sequence of dependent trials. Biometrika **46** (1959), 454–460.

A sequence of trials is represented as a Markov chain with two states, "success" and "failure", and stationary transition probabilities. The distribution of the number S of successes in the first n trials is obtained and the moments of S up to order four are given. Approximations for the moments are also considered.

W. Hoeffding (Chapel Hill, N.C.)

250:

Hennequin, Paul-Louis. Semi-groupes généralisés et processus de Markoff non homogènes. C. R. Acad. Sci. Paris **249** (1959), 1736–1737.

Given a Markov process with state space E, sample paths $t\to x(t)\in E$, and transition probabilities $P(x(t_2)\in db|x(t_1)=a)$ depending upon t_1, t_2 ($\geqq t_1$), a, and b, the space-time motion $t\to\dot{x}(t)=[t,x(t)]\in[0,+\infty)\times E$ is Markovian and its transition probabilities $P(\dot{x}(t_2)\in d\dot{b}|\dot{x}(t_1)=\dot{a})$ depend upon the points $\dot{a}$ and $\dot{b}$ of the augmented state space $\dot{E}=[0,+\infty)\times E$ and on the difference t_2-t_1; in brief, the law of the motion can be simplified in its dependence on time at the expense of adding an extra coordinate to the state space. The author applies a modification of this remark to the description of Markov processes with isolated jumps in terms of the conditional laws $P(\mathfrak{m}>t, x(t_2)\in db|x(t_1)=a)$ and $P(\mathfrak{m}\in dt_2,\ x(\mathfrak{m})\in db|x(t_1)=a)$, where $\mathfrak{m}$ is the first jump time (see J. E. Moyal [Acta Math. **98** (1957), 221–264; MR **20** #344] for background and more information).

H. P. McKean, Jr. (Cambridge, Mass.)

251:

Kawata, T. Some convergence theorems for stationary stochastic processes. Ann. Math. Statist. **30** (1959), 1192–1214.

Let $\mathscr{E}(t)$ be a continuous parameter wide sense stationary stochastic process. Let $f(t)$ and $K(t)$ be numerical valued functions. Let $X(t)=f(t)+\mathscr{E}(t)$. In the first half of the paper it is shown that the formulas

$$E\left|n\int_{-\infty}^{\infty}X(t-s)K(ns)ds-X(t)\int_{-\infty}^{\infty}K(s)ds\right|^2=o(1/n),$$

$$\text{l.i.m.}_{a\to 0}\int_{-\infty}^{\infty}X(t)e^{-ixt}\,aK(at)dt=\text{l.i.m.}_{T\to\infty}(2T)^{-1}\int_{-T}^{T}X(t)e^{ixt}dt\int_{-\infty}^{\infty}K(t)dt$$

hold under conditions which are extensions of conditions given by Bochner in the case that $X(t)=f(t)$. The second half of the paper studies some properties of the periodogram

$$J(T)=(4\pi T)^{-1}\left|\int_{-T}^{T}\mathscr{E}(t)e^{ixt}dt\right|^2$$

in the case that $\mathscr{E}(t)$ is not assumed to be Gaussian but its fourth cumulant function satisfies certain conditions.

E. Parzen (Stanford, Calif.)

252:

Malécot, G. Le problème du mouvement Brownien et ses généralisations. Publ. Inst. Statist. Univ. Paris **8** (1959), 67–75.

An expository article on certain random walk problems arising out of Brownian motion studies: steps of random length at random times, continuous random perturbation of particle velocity, etc.

S. Katz (New York)

253:

Kesten, Harry. Symmetric random walks on groups. Trans. Amer. Math. Soc. **92** (1959), 336–354.

A random walk on a countable group G generated by a set $A=\{a_1,a_2,\cdots\}$, $a_i\in G$, is defined as follows: Every step consists of right multiplication by a_i or a_i^{-1} both with

probability p_i. The set $P=\{p_1, p_2, \cdots\}$, $\sum p_i=1$, is called a probability distribution on the set A of generators and this distribution is said to be strictly positive if $p_i>0$ for all i. The random walk defines a Markov chain which can be represented by its matrix $M(G, A, P)$ of transition probabilities. The author wants to characterize the group G by the spectrum of $M(G, A, P)$ and gains interesting results in this direction; for instance the following invariance property: If the spectrum corresponding to the random walk on G, defined by a strictly positive probability distribution P on A, contains the value 1, then the spectrum corresponding to a random walk on any subgroup $H\subseteq G$ defined by any probability distribution Q on any set B of generators of H contains 1. It is also proved that such a spectrum contains 1 if G is a finite group or a countable abelian group. If the spectrum contains 1, then G has no free subgroups on more than one generator. Unsolved problems are discussed.

H. Bergström (Göteborg)

254:

Kaufmann, A; Cruon, R. Distribution du délai en régime transitoire dans une file d'attente. Chiffres **2** (1959), 253–266. (English, German and Russian summaries)

The distribution function $F_n(x)$ of waiting time of the nth arrival to a single server system, initially idle, with Poisson input at rate λ and exponential service time with service rate μ is determined, following F. Pollaczek [*Problèmes stochastiques* ..., Mémor. Sci. Math., no. 136, Gauthier-Villars, Paris, 1957; MR **19**, 987] in the following form:

$$F_n(x) = 1 - \sum_{k=0}^{n-2} [e^{-\mu x}(\mu x)^k/k!]a_{n,k}$$

with

$$a_{n,k} = \sum_{j=k}^{n-2} \rho^{j+1}(1+\rho)^{k-1-2j}\frac{k+1}{j+1}\binom{2j-k}{j}, \quad \rho = \lambda/\mu.$$

Numerical tables are given for the mean waiting times.

J. Riordan (New York)

255:

Pollaczek, Félix. Application de la théorie des probabilités à des problèmes posés par l'encombrement des réseaux téléphoniques. Ann. Télécommun. **14** (1959), 165–183.

This is a summary article, actually the development of lectures in a course given in 1956 at l'Institut de Statistique de la Faculté des Sciences de Paris, on the treatment in telephony of the simplest many-server system—the "fully accessible" case in which any idle server may attend any service demand. Both loss (busy signal) and delay systems are examined. For the former, the usual assumption is made that the calls finding all lines busy (which constitute the loss) are without effect on the traffic input; the story is enlivened by presenting a generating function for the probability that the nth call following an idle state is lost, assuming Poisson input and exponential service time, as well as the known stationary state probabilities (first Erlang distribution). For delay systems, the most interesting result is a new proof for the delay (waiting time) distribution function for Poisson input and constant service time, based on the remark that the servers may be numbered so that server i serves calls numbered $i+js$, $j=0, 1, \cdots$ (s is the number of servers), thus reducing the problem to that of a single server with Erlangian input. This reduction occurs naturally only for constant service time, but for general service distribution function is accomplished perforce by assigning calls to servers in sequence, a procedure followed in one of the author's earliest papers in the subject and one which gives added interest to the single server problem, at least for providing relatively simple bounds on the delays. Finally, the author's work on asymptotic formulas for both the probability of no delay and the mean delay for constant service time and on the stationary delay distribution for random service with Poisson input and exponential service time are resumed.

J. Riordan (New York)

256:

Ford, F. A. J. A note on the paper of Miller, Bernstein and Blumenson. Quart. Appl. Math. **17** (1959/60), 446.

Let X_i, $1\leq i\leq N$, be N independent Gaussian processes with means a_i and common autocovariance function. A generalized Rayleigh process R is defined as the square root of the sum of the squares of the X_i [Miller, Bernstein and Blumenson, Quart. Appl. Math. **16** (1958), 137–145; MR **20** #1371]. The author shows how the cumulative distribution function of R may be expressed in terms of tabulated functions.

K. S. Miller (New York)

257:

Gilbert, E. N.; Pollak, H. O. Amplitude distribution of shot noise. Bell System Tech. J. **39** (1960), 333–350.

Authors' summary: "A shot noise, $I(t)$, is a superposition of impulses occurring at random Poisson distributed times $\cdots, t_{-1}, t_0, t_1, t_2, \cdots$. In the simplest case, if the impulses all have the same shape $F(t)$, then $I(t)=\sum_i F(t-t_i)$. We study, in this and more general cases, the distribution function $Q(I)=\Pr[I(t)\leq I]$. One of our results is an integral equation for $Q(I)$. This yields explicit expressions for $Q(I)$ in a number of cases, including $F(t)=e^{-t}$; it also permits a computational technique which is applied to $F(t)=e^{-t}\sin\omega t$ for $\omega\gg 1$."

R. A. Silverman (New York)

258:

Лебедев, В. Л. ★Случайные процессы в электрических и механических системах. [Lebedev, V. L. Stochastic processes in electrical and mechanical systems.] Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1958. 176 pp. 6.45 rubles.

This little book is a practical introduction for engineers. Although dealing with a subject on which reams could be written, it is concise, selective, and assumes an elementary knowledge of probability, and of electrical/mechanical systems. An attempt at a rigorous measure-theoretic treatment would be out of place here, and none is made.

The first chapter gives a general account of the role of statistical theory in technology; the second introduces the concept of a random function, and various linear representations of random functions. The third chapter is devoted to stochastic influences on linear systems with constant lumped parameters, while one-dimensional Brownian motion, thermal noise in electrical conductors, and optimal filters are taken up in the fourth chapter. Problems involving random processes in nonlinear

systems are described briefly in Chapter V, while Chapter VI deals with various specific problems of electronic engineering; statistical properties of a detector output with signal plus noise input, etc.

V. E. Beneš (Murray Hill, N.J.)

259:

Bach, Günter. Über die Grössenverteilung von Kugelschnitten in durchsichtigen Schnitten endlicher Dicke. Mitt. Math. Sem. Univ. Giessen **57** (1959), iii + 33 pp.

In einer durchsichtigen Masse seien undurchsichtige Kugeln eingelagert, deren Radien eine Verteilung vom stetigen Typ mit Dichte G besitzen. Es wird ein Schnitt gegebener Dichte gemacht und senkrecht auf eine Ebene projiziert. Wegen der Undurchsichtigkeit der Kugeln wird hierbei der grösste Schnittkreis S abgebildet. Die Beziehung zwischen G und der Dichte g der Radien von S drückt sich als Volterrasche Integralgleichung zweiter Art aus (deren Herleitung stillschweigende Annahmen macht, z.B. eine Gleichverteilungshypothese für die Schnitte). Sie wird durch Iteration gelöst. Es werden Sonderfälle untersucht:

$$g(r) = \sum_{j=1}^{n} a_j r e^{-c_j r^2} \quad (a_j, c_j > 0),$$

$$g(r) = A r e^{-cr^2} \sum_{j=1}^{n} b_j L_j^{(\alpha)}(cr^2) \quad (A, c, b_j > 0),$$

$r \geqq 0$, wo $L_j^{(\alpha)}$ das $j$te Laguerrsche Polynom der Ordnung $\alpha \geqq 0$ ist, und in umgekehrter Richtung: G ist eine Dichte einer Pearson Typ III-Verteilung oder einer Rechteckverteilung. Nomogramme für die Auswertung der beiden letzten Beispiele sind beigefügt.

L. Schmetterer (Hamburg)

STATISTICS

See also A9, 624, 641, 647.

260:

Deming, Lola S. Selected bibliography of statistical literature, 1930 to 1957. I. Correlation and regression theory. J. Res. Nat. Bur. Standards Sect. B **64B** (1960), 55–68.

Author's summary: "This is the first in a series of bibliographies dealing with various specific subjects in the field of statistics. References and titles of important contributions in correlation and regression theory have been taken from technical journals published throughout the world since 1930."

261:

Deming, Lola S. Selected bibliography of statistical literature, 1930 to 1957. II. Time series. J. Res. Nat. Bur. Standards Sect. B. **64B** (1960), 69–76.

Author's summary: "This is the second in a series of bibliographies dealing with various specific subjects in the field of statistics. References and titles of important contributions to the study of time series have been taken from a wide variety of technical journals published in the many languages and countries which have been actively engaged in statistical analysis."

262:

Benedetti, Carlo. Di alcuni criteri di decisione statistica basati sulla soluzione di problemi di minima distanza. Metron **19**, no. 3–4, 22–37 (1959). (English summary)

An eminently practical algorithm is given for finding, for an arbitrary sequence $a = \{a_1, \cdots, a_n\}$ of real numbers, the sequence $\bar{x}$ that minimizes $\sum (a_i - x_i)^2$ among non-decreasing sequences x, thus. Let Sa be the sequence derived from a by replacing each element of each maximal, consecutive, non-increasing subsequence of a by the average over the subsequence. Since S decreases the range of a, when it makes any change at all, $S^k a$ is a fixed non-increasing $\bar{x}$ from some k on; this $\bar{x}$ is shown to solve the problem. Obviously k can never exceed $n-1$. Though this bound cannot be improved, k promises to be very small in many applications, some of which are discussed.

A paper by Clifford Hildreth [J. Amer. Statist. Assoc. **49** (1954), 598–619; MR **16**, 382] is interesting in connection with this one. *L. J. Savage* (Ann Arbor, Mich.)

263:

Pachares, J. Table of the upper 10% points of the studentized range (accompanied by revised tables of 5% and 1% points). Biometrika **46** (1959), 461–466.

264:

Bhattacharya, Nikhilesh. An extension of Hald's table for the one-sided censored normal distribution. Sankhyā **21** (1959), 377–380.

265:

Johnson, N. L. On an extension of the connexion between Poisson and χ^2 distributions. Biometrika **46** (1959), 352–363.

The mixture representation of non-central χ^2 and F distributions and the connexion of the title suggest approximations involving the normal distribution. Numerical comparisons include that with the approximation of the note following the paper [#266 below].

J. Hannan (East Lansing, Mich.)

266:

Pearson, E. S. Note on an approximation to the distribution of non-central χ^2. Biometrika **46** (1959), 364.

267:

Khatri, C. G. On certain properties of power-series distributions. Biometrika **46** (1959), 486–490.

Author's summary: "Noack [Ann. Math. Statist. **21** (1950), 127–132; MR **11**, 445] has defined a power-series distribution as $P(\xi = x) = a_x Z^x / f(Z)$ for $x = 0, 1, 2, \cdots$, where $a_x Z^x \geqq 0$, a_x is a function of x or constant, and $f(Z) = \sum_{x=0}^{\infty} a_x Z^x$ is convergent for $|Z| \leq r$. Here we establish the recurrence relations for cumulants and factorial cumulants, which are utilized to show that any power-series distribution in a single parameter is determined uniquely from its first two moments. The multivariate extensions of these results are shown with an illustration of multinomial distribution. All the above results extend to truncated power-series distributions."

L. A. Aroian (Los Angeles, Calif.)

268:
Pillai, K. C. S.; Tienzo, Benjamin P. On the distribution of the extreme studentized deviate from the sample mean. Biometrika **46** (1959), 467–472.

269:
Pillai, K. C. S. Upper percentage points of the extreme studentized deviate from the sample mean. Biometrika **46** (1959), 473–474.

270:
Weibull, Christer. The distribution of the Student ratio in the case of serially correlated normal variables. Skand. Aktuarietidskr. **1958**, 172–176 (1959).

If x is N-normal $(0, V)$, $N^{1/2}\bar{x}$ and $\sum (x_j-\bar{x})^2$ are, if (and only if) $\bar{x}$ and $x-\bar{x}1$ are uncorrelated, representable as $\lambda_N^{1/2}z_N$ and $\sum_1^{N-1}\lambda_j z_j^2$, where the λ_j are the roots of V with $N\lambda_N = 1'V1$ and the z_j are independent standard normals.

The paper considers only $V=AA'$ with $a_{\mu\nu}=a_{\nu-\mu(N)}$, where $a_0, \cdots, a_{N-1}$ are given reals with $\sum a_j \neq 0$. Then

$$\lambda_j = \sum\sum a_\mu a_\nu \cos 2\pi(\mu-\nu)j/N = \lambda_{N-j}$$

and, for N odd, the distribution of the t-statistic is a finite scale mixture of $t(2)$-distributions (cf. Theorem 2.4 of Box [Ann. Math. Statist. **25** (1954), 290–302; MR **15**, 884]). The statements on the non-existence of moments and on the asymptotic variance are notably erroneous; $E|t|^k < \infty$ if and only if $k+1 <$ rank V and, under weak boundedness conditions on the a_j, t is asymptotically normal $(0, (\sum a_j)^2/\sum a_j^2)$.

J. Hannan (East Lansing, Mich.)

271:
Weiler, H. Means and standard deviations of a truncated normal bivariate distribution. Austral. J. Statist. **1** (1959), 73–81.

272:
Dwass, Meyer. The distribution of a generalized D_n^+ statistic. Ann. Math. Statist. **30** (1959), 1024–1028.

Let $F_n(x)$ be the sample c.d.f. for a sample of size n from the continuous population c.d.f. $F(x)$. Elementary derivations are given of the exact distribution of $\sup_{-\infty \leq x \leq \infty} [F_n(x) - cF(x)]$, where $c > 0$.

I. R. Savage (Minneapolis, Minn.)

273:
Fisz, M. Characterization of some probability distributions. Skand. Aktuarietidskr. **1958**, 65–67 (1959).

Let x_1, x_2 be independent random variables with a common distribution function $F(x)$. Let y_1, y_2 denote the values of x arranged in increasing order: $y_1 < y_2$. Denote $z_1 = y_1/y_2$, $z_2 = y_2$. Let $F(x)$ be absolutely continuous with $F(0) = 0$. Then

$$f(x) = ((m+1)/b^{m+1})x^m \quad (0 \leq x \leq b),$$
$$= 0 \quad (0 > x, x > b)$$

is the density function of $F(x)$ if and only if the random variables z_1, z_2 are independent. This is the generalisation of a result by the reviewer [Skand. Aktuarietidskr. **33** (1950), 214–222; MR **12**, 725].

S. Malmquist (Stockholm)

274:
Krishna Iyer, P. V. A simplified method for evaluating the product cumulants of some distributions arising from a sequence of observations. J. Indian Soc. Agric. Statist. **10** (1958), 33–40, xvi. (Hindi summary)

Sequences of n, each member of which may be $A, B, C, \cdots$ with respective probabilities $p, q, r, \cdots$, are classified according to the number of joins (consecutive pairs) of some specified kind, like $AA, AB, AC, \cdots$, or jointly with respect to the number of joins of each of the specified kinds, like AA and AB, AA and AC, $\cdots$. The product cumulants for joins of two kinds are calculated by an expansion into terms corresponding to products of partitions of r and s, the cumulant order numbers. Cumulants K_{31} and K_{22} for joins AA and AB in a sequence of n are exhibited in detail in two tables, the first of which also includes the summary expression for K_{13}.

J. Riordan (New York)

275:
Blalock, H. M., Jr. Probabilistic interpretations for the mean square contingency. J. Amer. Statist. Assoc. **53** (1958), 102–105.

In a two-way classification of individuals in a finite population, the mean square contingency ϕ^2, under the proportional prediction model in which one compares the probabilities of errors made in assigning an individual to the proper B-class with and without knowledge of the proper A-class, can be interpreted as the difference between the sum of probabilities of error of not knowing A and the sum of probabilities of error given the A-class. Thus, ϕ^2 is a function of probabilities, each of which can be given a simple interpretation. However, these interpretations indicate the arbitrary nature of the measure. The measure τ_b, as defined by Goodman and Kruskal, can be interpreted as the relative decrease in the proportion of incorrect predictions over the whole table when information about A is used. Therefore, it appears that τ_b is generally to be preferred over ϕ^2 or other measures based on chi-square.

D. Teichroew (Stanford, Calif.)

276:
Cohen, A. Clifford, Jr. Misclassified data from a binomial population. Technometrics **2** (1960), 109–113.

Author's summary: "The method of maximum likelihood is employed to estimate the binomial parameter p whenever an erroneous observation or report results in recording c defective samples when there are actually $c+1$. The proportion of such erroneous observations is also estimated, and asymptotic variances and covariances of the estimates are obtained. A numerical example is included."

277:
Calleja Siero, Anselmo. Samples from finite populations: Methods of estimation. Estadíst. Española no. 4, 30–48 (1959). (Spanish. English summary)

278:

Stein, Charles. The admissibility of Pitman's estimator of a single location parameter. Ann. Math. Statist. **30** (1959), 970–979.

Suppose $X_1, \cdots, X_n$ are independent and identically distributed random variables, each with density function $r(x-\xi)$, where ξ is an unknown location parameter. Pitman showed that the estimator

$$\hat{\xi}_0(X_1, \cdots, X_n) = \frac{\int \xi r(X_1-\xi)\cdots r(X_n-\xi)d\xi}{\int r(X_1-\xi)\cdots r(X_n-\xi)d\xi}$$

is the "best translation-invariant" estimator [Biometrika **30** (1939), 391–421]. The author shows that $\hat{\xi}_0$ is admissible, under certain conditions. That is to say, if

$$E[(\hat{\xi}-\xi)^2|\xi] \leq E[(\hat{\xi}_0-\xi)^2|\xi]$$

for all ξ, then actually equality holds for all ξ. (Compare Karlin's paper, Ann. Math. Statist. **29** (1958), 406–436 for a similar result under somewhat more special conditions.) *M. Dwass* (Evanston, Ill.)

279:

Kupperman, Morton. On comparing two observed frequency counts. Appl. Statist. **9** (1960), 37–42.

Author's summary: "A simplified method of calculating the χ^2 statistic for deciding whether or not two sample frequency counts came from the same unspecified population is discussed. An alternative statistic is presented; it is based on information-theory considerations and has the advantage of being simpler to calculate."

280:

Chatelet, F.; Lansraux, G.; Million, B. Tables pour le calcul approché des intégrales finies de Hankel. Chiffres **2** (1959), 233–237. (English, German and Russian summaries)

The computation of the integral

$$G(y) = \int_0^1 F(x)J_0(xy)d(x^2)$$

is effected by replacing $F(x)$ by its approximate Lagrangian interpolation polynomial $F(x) \doteq \sum_{i=1}^{N} P_i(x)F(x_i)$, so that $G(y) \doteq \sum F(x_i)\mathscr{L}_i^N(y)$, where

$$\mathscr{L}_i^N(y) = \int_0^1 P_i(x)J_0(xy)d(x^2).$$

Ten-decimal values are given of $\mathscr{L}_i^N(y)$ for $N=11$, $i=1(1)11$, $y=0(1)12$, which give better accuracy than the use of previous tables for $N=6$ by G. Lansraux [Rev. Opt. **26** (1947), 278–294; MR **9**, 317; p. 286]. It is convenient in practical applications to have equal intervals in x_i^2, so that in each case $(N-1)x_i^2 = i-1$, for $i=1(1)N-1$. *L. Fox* (Oxford)

281:

DeGroot, M. H.; Nadler, Jack. Some aspects of the use of the sequential probability ratio test. J. Amer. Statist. Assoc. **53** (1958), 187–199.

282:

Steel, Robert G. D. A multiple comparison sign test: treatments versus control. J. Amer. Statist. Assoc. **54** (1959), 767–775.

Author's summary: "Let $(X_{0j}, X_{1j}, \cdots, X_{kj})$ be the result of a single trial, where the subscript 0 is associated with a control and the subscripts $1, \cdots, k$ with treatments. To test the joint hypothesis $P(X_{ij}-X_{0j}<0) = 1/2 = P(X_{ij}-X_{0j}>0)$, all i, compute the test criterion $(r_1, \cdots, r_k)$ where r_i is the number of times $X_{ij}-X_{0j}$ is negative in n trials. A method of computing the distribution of $(r_1, \cdots, r_k)$ is illustrated. Exact probability distributions of $\min r_i$ are given for $k=2$, $n=4(1)10$ and $k=3$, $n=4(1)7$. It is conjectured that $2(\min r - n/2)/\sqrt{n}$ is distributed approximately as Dunnett's t. Tables based on this conjecture are computed and values are seen to agree well with comparable values from the exact distribution."

H. P. Edmundson (Pacific Palisades, Calif.)

283:

Dunn, Olive Jean. Confidence intervals for the means of dependent, normally distributed variables. J. Amer. Statist. Assoc. **54** (1959), 613–621.

An expository paper providing tables and a worked example for some of the procedures previously presented by the author [Ann. Math. Statist. **29** (1958), 1095–1111; MR **21** #399]. (It should be noted that the case of variances unknown but equal can be generalized directly to the case of variances unknown but with known ratios.) Errata: P. 616, line 11-b, insert "the last row of" before "Table 615". P. 616, line 8, insert "point" before "of the same distribution". *P. Meier* (Chicago, Ill.)

284:

Chapman, Douglas G. A comparative study of several one-sided goodness-of-fit tests. Ann. Math. Statist. **29** (1958), 655–674.

The paper is devoted to a comparative study of distribution-free tests of the simple hypothesis H_0: $F=F_0$ against the composite alternative: $F<F_0$. Let $\varphi(X_1, \cdots, X_n)$ be such a test of size α, i.e., a measurable function from R_n to (0, 1) such that $E(\varphi|F_0) \leq \alpha$, and let $E(\varphi|F)$ be its power function. The author introduces two properties of a test: (1) φ is monotone if $x_i \geq y_i$, $i=1, \cdots, n$, implies $\varphi(x_1, \cdots, x_n) \geq \varphi(y_1, \cdots, y_n)$ for all values x_i, y_i; (2) φ is partially ordered if $F_1 \leq F_2$ always implies $E(\varphi|F_1) \geq E(\varphi|F_2)$. He observes that if φ is continuous, except for a finite number of jumps, and partially ordered then it is unbiased; and that if φ is of structure (d) and monotone then it is partially ordered. Denoting

$$\sup_{(x)} [F(x)-G(x)] = \rho^-(F, G),$$

he then undertakes to find sharp upper and lower bounds for the power of partially ordered tests of H_0 against alternatives $F<F_0$ such that $\rho^-(F_0, F)=\Delta$ is given. This study is carried out for several specific distribution-free one-sided tests of fit and yields a number of theoretical findings as well as interesting tables of numerical comparisons. *Z. W. Birnbaum* (Seattle, Wash.)

285:

Guttman, Irwin. Optimum tolerance regions and power when sampling from some non-normal universes. Ann. Math. Statist. **30** (1959), 926–938.

The methods of Fraser and Guttman [same Ann. **27** (1956), 162–179; MR **17**, 871] are applied to the single and double exponential distribution.

I. R. Savage (Minneapolis, Minn.)

286:

Cowden, Dudley J. A procedure for computing regression coefficients. J. Amer. Statist. Assoc. **53** (1958), 144–150.

The author describes a method for computing regression coefficients from regression coefficients of lower order. No references to comparable results of earlier authors are given.

P. S. Dwyer (Ann Arbor, Mich.)

287:

Foote, Richard J. A modified Doolittle approach for multiple and partial correlation and regression. J. Amer. Statist. Assoc. **53** (1958), 133–143.

The author presents a Doolittle method for obtaining multiple and partial correlation and regression coefficients (with a desk calculator) which features the use of the inverse of the moment matrix.

P. S. Dwyer (Ann Arbor, Mich.)

288:

Sen, Pranab Kumar. On the moments of the sample quantiles. Calcutta Statist. Assoc. Bull. **9**, 1–19 (1959).

Sufficient conditions are given for the existence of a moment of arbitrary order of a sample quantile and for the asymptotic equality of a moment of a sample quantile to the corresponding moment of a normal distribution. Other related problems are also considered.

W. Hoeffding (Chapel Hill, N.C.)

289:

Savage, I. Richard. Contributions to the theory of rank order statistics—the one-sample case. Ann. Math. Statist. **30** (1959), 1018–1023.

Suppose that $X_1, \cdots, X_n$ are n independent and identically distributed random variables and $|X_{i_1}| < |X_{i_2}| < \cdots < |X_{i_n}|$ is the ordering of the absolute values. (The X's are assumed to have continuous c.d.f.'s). The random vector $Z = (Z_1, \cdots, Z_n)$ is defined by $Z_k = 1$ if X_{i_k} is positive and $Z_k = 0$ otherwise. Tests based on Z are often used to test the hypothesis that the X's are symmetrically distributed about the origin. (When the hypothesis prevails, the 2^n possible values of Z are equally likely.) For the purpose of finding powerful tests against certain alternatives, it is shown that under certain conditions, $P(Z=z) > P(Z=z')$ where z is derived from z' be replacing a 0 by a 1, or interchanging a 0 and 1 in z' by moving the 1 to the right. The conditions require that the X's follow a certain monotone-likelihood, exponential law, which includes the normal distribution.

M. Dwass (Evanston, Ill.)

290:

Rogers, Gerald S. A note on the stochastic independence of functions of order statistics. Ann. Math. Statist. **30** (1959), 1263–1264.

"Theorem: Let x be a real random variable with distribution function $F(x) = \int_{-\infty}^{x} f(X)dX$, where $f(x)$ is a non-degenerate probability density function (pdf). Let $x_1 \leqq x_2 \leqq \cdots \leqq x_n$ be the order statistics based on a random sample of size $n \geqq 2$ from this x distribution. Let $z = z(x_1, \cdots, x_j)$ be a statistic based on the first $j < n$ order statistics only. Then the following two statements are equivalent: (a) z is independent of x_k for some $k \geqq j$; (b) z is independent of the set $\{x_k : j \leqq k \leqq n\}$." Applications are not presented.

I. R. Savage (Minneapolis, Minn.)

291:

Walsh, John E. Nonparametric mean and variance estimation from truncated data. Skand. Aktuarietidskr. **1958**, 125–130 (1959).

From the author's summary: "This paper considers situations where a known number of the smallest values of a sample and a known number of the largest values have been truncated. The problem is to obtain an estimate of the population mean, an estimate of the standard deviation of this estimate of the mean, and an estimate of the population standard deviation. This paper derives a nonparametric estimate for each of these three cases."

Z. W. Birnbaum (Seattle, Wash.)

292:

Cohen, Arthur. Table for the sign test when observations are estimates of binomial parameters. J. Amer. Statist. Assoc. **54** (1959), 784–793.

A modified sign test is proposed for testing the hypothesis that in k independent pairs of observations from binomial distributions with parameters (m_i, p_{xi}) and (n_i, p_{yi}) we have $p_{xi} = p_{yi}$, $i = 1, \cdots, k$ (m_i and n_i known). The test makes use of a theorem by the reviewer [Ann. Math. Statist. **27** (1956), 713–721; MR **18**, 240].

W. Hoeffding (Chapel Hill, N.C.)

293:

Hudimoto, Hirosi. On a two-sample non-parametric test in the case that ties are present. Ann. Inst. Statist. Math. Tokyo **11** (1959), 113–120.

Working with discrete distributions a relationship is developed between the distributions of the Wilcoxon and Kolmogorov-Smirnov test statistics. [See Z. W. Birnbaum and R. C. McCarty, Ann. Math. Statist. **29** (1958), 558–562; MR **20** #394.]

I. R. Savage (Minneapolis, Minn.)

294:

Walsh, John E. Nonparametric properties of some maximum likelihood estimates for median of symmetrical population. Ann. Inst. Statist. Math. Tokyo **11** (1959), 81–88.

Assume the available data from a sample is restricted to symmetrical pairs of order statistics and that a specified symmetric parametric family of distributions had been sampled. Then the maximum likelihood estimate of the median will be translation invariant and symmetrically distributed no matter what symmetrical population was sampled.

I. R. Savage (Minneapolis, Minn.)

295:
Kramer, Clyde Young; Glass, Suzanne. Analysis of variance of a Latin square design with missing observations. Appl. Statist. **9** (1960), 43–50.

Authors' summary: "The authors here describe a general method for estimating several missing values and analysing the results so obtained. They give explicit formulae for all cases when two values are missing, formulae for some other specific cases, and an illustrative example."

296:
Shah, B. V. A generalisation of partially balanced incomplete block designs. Ann. Math. Statist. **30** (1959), 1041–1050.

The conditions for an incomplete block design to be partially balanced (PBIB) are given by, e.g., Bose and Shimamoto [J. Amer. Statist. Assoc. **47** (1952), 151–184; MR **14**, 67]. The author first generalizes the relation connecting two treatments which are ith associates and then provides a further generalization resulting from the partitioning of the treatments into several groups. The generalized designs possess least squares analyses that are "almost similar to that of a PBIB."

H. L. Seal (New Haven, Conn.)

297:
Vartak, Manohar Narhar. The non-existence of certain PBIB designs. Ann. Math. Statist. **30** (1959), 1051–1062.

Extension of results of Shrikhande [same Ann. **21** (1950), 106–111; MR **11**, 306] to PBIB designs with three associate classes in which the restrictions $p_{11}^1 = n_1 - 1 = v_2 - 2$; $p_{22}^2 = n_2 - 1 = v_1 - 2$; $n_3 = n_1 n_2$ are superimposed on the usual Bose and Shimamoto [see reference in foregoing review] parametric definitions.

H. L. Seal (New Haven, Conn.)

298:
Ogawa, Junjiro. A necessary condition for existence of regular and symmetrical experimental designs of triangular type, with partially balanced incomplete blocks. Ann. Math. Statist. **30** (1959), 1063–1071.

PBIB designs with two associate classes are called triangular if any two treatments which are ith associates ($i = 1, 2$) occur together in λ_i blocks. Following Shrikhande [see reference in preceding review] a necessary condition is given for the existence of such a design in terms of the Hasse-Minkowski p-invariant.

H. L. Seal (New Haven, Conn.)

299:
Bailey, Norman T. J. The use of linear algebra in deriving prime power factorial designs with confounding and fractional replication. Sankhyā **21** (1959), 345–354.

From the author's summary: "This paper discusses the derivation of prime power factorial designs, with confounding and fractional replication, using only comparatively elementary results in linear algebra. Some further simplifications and systematization of the standard theory of factorial designs are given. The procedures recommended are also extremely quick in practice, and are very easily understood and acquired by students."

J. Kiefer (Ithaca, N.Y.)

300:
Singh, Daljit. Tables of orthogonal polynomials when the independent variable 'x' is in the geometric progression: $x_r = 2^{r-2}$; $x_1 = 0$; $x_r = 2^{r-1}$; $x_1 = 1$. J. Indian Soc. Agric. Statist. **10** (1958), 131–140.

Tables for 2 sets of orthogonal polynomials, useful in certain biological and agronomic experiments, are presented for values of x in geometric progression. In the first set $x_1 = 0$ and in the second set $x_1 = 1$. Polynomials are presented through the fourth degree for $n = 3, 4, 5$ and through the third degree for $n = 6, 7, 8$.

P. S. Dwyer (Ann Arbor, Mich.)

301:
Jones, R. Morley. On a property of incomplete blocks. J. Roy. Statist. Soc. Ser. B **21** (1959), 172–179.

The conditions are determined which an incomplete block design must satisfy so that, for any two orthogonal treatment contrasts, the intra- and inter-block components of these contrasts are all mutually orthogonal, and it is shown that, in addition to balanced incomplete block designs, there exist designs with blocks of unequal size which also have this property. Also, the conditions under which a given contrast has components with expected values proportional to that of the original contrast are determined.

A matrix, M, closely related to the incidence matrix for the design is found to play a central role in the investigation of these conditions, and the properties of M are exploited to give a simple proof of a theorem due to Yates concerning the efficiency of balanced designs.

P. Meier (Chicago, Ill.)

302:
Kapur, M. N. On a class of decision procedures for ranking normal populations according to their variances. J. Indian Soc. Agric. Statist. **10** (1958), 99–106, xx. (Hindi summary)

303:
Cox, D. R. A renewal problem with bulk ordering of components. J. Roy. Statist. Soc. Ser. B **21** (1959), 180–189.

Author's summary: "Suppose that k components are in use simultaneously, that batches of n items are ordered, and that all items in use at one time must be from the same batch. After $n-k+1$ failures have occurred the remaining $k-1$ items are wasted. Properties of the system are investigated and different strategies of use compared."

S. W. Nash (Ames, Iowa)

304:
Mandel, John. The measuring process. Technometrics **1** (1959), 251–267.

An informed and ramified theoretical discussion is given of the sort of experiment in which each of several laboratories measures (or assays) each of several specimens several times. *L. J. Savage* (Ann Arbor, Mich.)

305:
Hosford, John E. Measures of dependability. Operations Res. **8** (1960), 53–64.

The author considers repairable systems which are

subjected to a pattern of on (duty) and off cycles. The system can at any moment be in either of two states, "operable" or "failed". Failure and repair times are assumed to be exponentially distributed. For such a system steady state results are obtained for the following three measures of "dependability": (1) "pointwise availability"—the probability that the system will be operable at the start of a duty interval; (2) "reliability"—the probability that the system will not fail during a duty interval; (3) "interval availability"—the expected fraction of time that the system is operable during a duty interval. *Benjamin Epstein* (Stanford, Calif.)

306:

Azorín Poch, Francisco. Sampling of finite populations. Systematic sampling. Estadíst. Española No. **5**, 34–41 (1959). (Spanish. French summary)

An elementary "text-book" exposition of the theory of systematic sampling.

T. N. E. Greville (Kensington, Md.)

307:

Tikkiwal, B. D. An examination of the effect of matched sampling on the efficiency of estimators in the theory of successive sampling. J. Indian Soc. Agric. Statist. **10** (1958), 16–22, xv. (Hindi summary)

When numerical characteristics of a population are to be estimated at various times, and some of the units sampled at each time are units sampled at previous times, the author shows that the variance of the estimates decreases with time. *L. Weiss* (Ithaca, N.Y.)

308:

Bartlett, M. S. The impact of stochastic process theory on statistics. Probability and statistics: The Harald Cramér volume (edited by Ulf Grenander), pp. 39–49. Almqvist & Wiksell, Stockholm; John Wiley & Sons, New York; 1959. 434 pp. $12.50.

The author discusses the influence that the theory of stochastic processes has had on statistics historically. Time series analysis and the statistical analysis of point processes are two classes of statistical problems that are mentioned in particular.

M. Rosenblatt (Providence, R.I.)

309:

Tukey, John W. An introduction to the measurement of spectra. Probability and statistics: The Harald Cramér volume (edited by Ulf Grenander), pp. 300–330. Almqvist & Wiksell, Stockholm; John Wiley & Sons, New York; 1959. 434 pp. $12.50.

This paper is for the most part concerned with the practical aspects of obtaining estimates of the spectra of stationary time series. Much space is devoted to discussion of the need for "prewhitening" (a term and a technique introduced by the author) the data. The difficulties of "aliasing" when dealing with discretely sampled time series are also considered. There are brief references to higher order spectra and spectral analysis for multivariate time series.

M. Rosenblatt (Providence, R.I.)

310:

Lomnicki, Z. A.; Zaremba, S. K. Bandwidth and resolvability in statistical spectral analysis. J. Roy. Statist. Soc. Ser. B **21** (1959), 169–171.

In the theory of statistical spectral analysis of time series, estimates have been evaluated on the basis of one or more of the following properties: (i) resolvability, (ii) statistical reliability, (iii) bandwidth, (iv) bias, (v) variance, (vi) mean square error. The authors of the present note point out that the published controversy between them [same J. **19** (1957), 13–37 (discussion 47–63); MR **19**, 1098] and U. Grenander [same J. **20** (1958), 152–157; MR **20** #2827] is due to a lack of agreement on the meanings of the first three concepts listed. The authors contend that (i) "resolvability, in the generally accepted sense of ability to discriminate between spectral densities at neighboring frequencies, cannot be identified with narrowness of bandwidth," and (ii) statistical reliability depends not only on variance but also on bias.

E. Parzen (Stanford, Calif.)

NUMERICAL METHODS

See also 280, 449.

311:

Ehrmann, Hans. Iterationsverfahren mit veränderlichen Operatoren. Arch. Rational Mech. Anal. **4**, 45–64 (1959).

Es werden Iterationsverfahren $u_{n+1}=T_nu_n$ zur Lösung einer Gleichung $u=Tu$ in einem abstrakten Raum R untersucht. In R sei ein Abstand $\rho(u, v)$ definiert, der Element eines halbgeordneten linearen Raumes N ist. Die in $D\subset R$ erklärten Operatoren T_n sollen in bestimmtem Sinne gegen T konvergieren und in einem beschränkten Gebiet $G\subset D$ gleichmässig lipschitzbeschränkt sein: $\rho(T_nu, T_nv)\leq P\rho(u, v)$ mit einem stetigen, linearen, positiven Operator P. Die Reihe $\sum_{\nu=0}^{\infty}P^\nu\rho$ konvergiere für $\rho\in N$. G enthalte u_0 und alle Punkte $u\in R$ mit

$$\rho(u, u_1)\leq(E-P)^{-1}P\rho(u_0, u_1)+(E-P)^{-1}\tau_0$$

bei $\tau_0\geq(T_nu_0, T_0u_0)$. Ausser Existenz-, Eindeutigkeits- und Konvergenzaussagen wird eine Fehlerabschätzung

$$\rho(u, u_1)\leq(E-P)^{-1}P\rho(u_0, u_1)+(E-P)^{-1}\rho(Tu_0, u_1)$$

bewiesen. Die Konvergenzforderung $T_n\to T$ kann wegfallen, wenn auch T durch P lipschitzbeschränkt ist und auch $\tau_0\geq(Tu_0, T_0u_0)$ gilt. Die Ergebnisse werden auf gewöhnliche nichtlineare Gleichungen, Systeme nichtlinearer Gleichungen und nichtlineare Randwertaufgaben angewendet. Vier numerische Beispiele zeigen die Nützlichkeit der Methode. *J. Schröder* (Hamburg)

312:

Wynn, P. A sufficient condition for the instability of the q-d algorithm. Numer. Math. **1** (1959), 203–207.

The qd algorithm applied to transforming a power series $\sum_0^\infty c_k/z^{k+1}$ into a continued fraction $\cfrac{C_0}{|z-\alpha_1}-\cfrac{\beta_1}{|z-\alpha_2}-\cdots$ tends to be unstable. This is not astonishing since the coefficients α, β of the continued fractions may

be extremely sensitive to changes of the coefficients c_k of the power series. Since the author investigates solely the errors of the α, β for a given error distribution of the c_k without considering errors produced by intermediate qd-steps, his error analysis pertains to the α, β as functions of the c_k rather than to the qd-algorithm. Furthermore the results are very special because the author considers only the error distribution $\delta c_k = \varepsilon c_{k+1}$.

H. Rutishauser (Zürich)

313:

Floyd, Robert W. A note on rational approximation. Math. Comput. **14** (1960), 72–73.

314:

Stoller, David S. Fitting position data to minimize velocity errors. SIAM Rev. **2** (1960), 31–35.

From the author's introduction: "This paper gives a solution to the following problem. An object in rectilinear motion is observed over a given time interval. Position measurements are made during the time interval. It is desired to estimate the velocity of the object so as to minimize the velocity error."

315:

Clenshaw, C. W. Curve fitting with a digital computer. Comput. J. **2** (1959/60), 170–173.

G. Forsythe [J. Soc. Indust. Appl. Math. **5** (1957), 74–88; MR **19**, 1079] described a method for fitting a polynomial $Y(x)$ to a set of points (x_r, y_r) $(r=1, \cdots, n)$ using the principle of least squares. In this method, $Y(x)$ is expanded in a series of orthogonal polynomials $p_i(x)$ $(i=1, \cdots, m)$ in order to overcome the usual problems of illconditioning. The present paper describes in detail a modification of Forsythe's method with the aim of reducing storage requirements. This is achieved by representing each polynomial $p_i(x)$ within the computer by the coefficients of its Chebyshev expansion. In comparison the original method assumes that the values $p_i(x_r)$ $(r=1, \cdots, n)$ are stored in the computer. The use of Chebyshev polynomials avoids certain possibilities of loss of accuracy usually connected with the explicit representation of polynomials in terms of powers of x. For machines with small high-speed storage the modified method might be quite advantageous although Forsythe's original method may possibly be faster especially when polynomials of high degree are required.

W. C. Rheinboldt (Syracuse, N.Y.)

316:

Cheney, E. W.; Goldstein, A. A. Newton's method for convex programming and Tchebycheff approximation. Numer. Math. **1** (1959), 253–268.

This paper gives a number of related computational algorithms for the problem of minimizing the function of n variables $F(x)=\sup_i \sum_{j=1}^n A_j^i x_j - b_i$, where $\{A_j^i\}$ and $\{b_i\}$ are given constants $(i=1, \cdots, m, j=1, \cdots, n)$. While differing from one another in details, the six algorithms discussed follow this general pattern. Let $R^i(x)=\sum_{j=1}^n A_j^i x_j - b_i$, $i \in M=\{1, 2, \cdots, m\}$. For suitable $I \subseteq M$, a unique point x^k is defined by $R^i(x^k)=0$ for all $i \in I$; a simple test on the data of the R^i shows whether x^k minimizes the function $F_I(x)=\sup_{i \in I} R^i(x)$. Assuming x^k to have this property, the set I is augmented by i° such that $R^{i^\circ}(x^k)>0$, and i' is found such that, with $I'=I \cup \{i^\circ\}-\{i'\}$, one has a new point x^{k+1} with $R^i(x^{k+1})=0$ for $i \in I'$, x^{k+1} minimizing $F_{I'}(x)$, and $F_{I'}(x^{k+1}) \geq F_I(x^k)$. (More precisely, this is the Algorithm I of the paper.)

The algorithms are shown to yield a sequence $\{x^k\}$ such that, with I as defined above, $F_I(x^k)$ converges monotonically from below to $\inf_x F(x)$. A detailed flow diagram is given for one of the algorithms, and some material bearing on the approximation of a given continuous function on a compact metric space by others.

P. Wolfe (Santa Monica, Calif.)

317:

Rutishauser, Heinz. Deflation bei Bandmatrizen. Z. Angew. Math. Phys. **10** (1959), 314–319. (English summary)

Der Verfasser gibt ein Verfahren an, das aus einer symmetrischen matrix A n-ter Ordnung mit einem bekannten Eigenwert λ und dem zugehörigen (auch bekannten) Eigenvektor v eine neue symmetrische Matrix A_1 $(n-1)$-ter Ordnung mit denselben Eigenwerten wie A (mit Ausnahme von λ) entstehen lässt. Während bei anderen bekannten Methoden die sog. Bandgestalt der Matrix $A=(a_{ik})$, d.h., die Eigenschaft, dass $a_{ik}=0$ für $|i-k|>m$ gilt, allgemein verlorengeht, bleibt diese (mit demselben m) bei dem vorliegenden Verfahren erhalten. Die Hauptidee liegt in einer zu v zugeordneten Orthogonaltransformation, deren Matrix H die sog. Hessenberg-Gestalt (d.h., $h_{ik}=0$ für $i>k+1$) hat, wobei H^TAH direkte Summe von A_1 und λ ist.

M. Fiedler (Prague)

318:

Haynsworth, Emilie V. Applications of a theorem on partitioned matrices. J. Res. Nat. Bur. Standards Sect. B **63B** (1959), 73–78.

The matrices are of the form $A=(A_{ij})$, where each A_{ij} is itself a matrix and for $i=j$ is square. If every A_{ij} is of the same order and lower triangular, A is reducible. If A is block-stochastic (for each block the row sums are equal), the characteristic roots are those of the matrix of the row sums, and of another matrix easily written down. Other cases are described permitting analogous reductions. An example arising in molecular physics is given.

A. S. Householder (Oak Ridge, Tenn.)

319:

Schröder, Johann. Anwendung von Fixpunktsätzen bei der numerischen Behandlung nichtlinearer Gleichungen in halbgeordneten Räumen. Arch. Rational Mech. Anal. **4**, 177–192 (1959).

Let R be a partially ordered linear topological space over the real domain. Consider an equation of the form $u=Tu$ where T is a linear or non-linear operator which maps R into itself. For the numerical treatment of such equations the contraction mapping theorem has been used in various forms. In several previous papers [see e.g., J. Schröder, same Arch. **3** (1959), 28–44; MR **21** #1689; L. Collatz and J. Schröder, Numer. Math. **1** (1959), 61–72; MR **21** #1688] the author and L. Collatz have shown that more general fixpoint-theorems, as for example the theorems of Brouwer, Schauder or Tychonoff, can also be efficiently used in numerical applications. This is

accomplished by the construction of sets $M \subset R$ such that the statement $u^* \in TM \subset M$ constitutes a numerically usable estimate for the solution u^* of $u = Tu$.

While in their previous papers the authors concerned themselves with special problems of this nature, the present paper consists of an interesting general theoretical discussion of this application of the general fixpoint theorems. The author considers not only equations of the form $u = Tu$, but also more generally, those of the form $Au = Bu$ where A and B are operators from R into itself which fulfill certain general conditions. It is further assumed that this more general equation can be reduced in a specified sense to an equation $u = Tu$. The sets M are chosen to be of the form $M = \{u \in R, x \leqq u \leqq y\}$ where $x, y \in R$ are given. A first theorem gives general conditions for x, y such that $TM \subset M$. Then for the special case of an equation $u = Tu$ a theorem is proven which contains as special case an earlier published theorem on monotonic operators [see MR **21** #1689]. For the iteration $u_{n+1} = Tu_n$, theorem 3 gives estimates for $u^* - u_0$ in terms of $u_1 - u_0$. Theorem 4 presents estimates for the error $u^* - u_0$ in terms of $-Au_0 + Bu_0$ where u_0 is any approximate solution of $Au = Bu$. For the purpose of illustrating the general theorems the author then makes some remarks on several special applications. Numerical examples are to follow in a later paper.

W. C. Rheinboldt (Syracuse, N.Y.)

320:

Pham, Daniel; Ghinea, Monique. Sur une méthode d'itération dans la théorie des équations. C. R. Acad. Sci. Paris **249** (1959), 2262–2264.

Les auteurs proposent pour l'équation $f(x) = 0$ des formules itératives écrites sous la forme de polynômes en f ou de fonctions homographiques. La plupart de ces formules sont déjà connues. *J. Kuntzmann* (Grenoble)

321:

Wilkinson, J. H. The evaluation of the zeros of ill-conditioned polynomials. I, II. Numer. Math. **1** (1959), 150–180.

It is known that Newton's rootfinding method (as well as the derived methods of Bairstow and Muller [see D. E. Muller, Math. Tables Aids Comput. **10** (1956), 208–215; MR **18**, 766]) may fail for polynomials of high order even if they have not very close roots. The reason for this is that in the region of an accumulation of roots, evaluation of the polynomial is subject to cancellations of digits and in addition the value of the polynomial is very sensitive to changes of the coefficients (for the latter reason any method for rootfinding will show the same kind of trouble).

In his papers, the author gives several typical numerical examples, as well as estimates for the error to be expected. The examples show that even for a polynomial with well separated real roots, the (single precision) computation may yield complex values for some of them. Of course this does not encourage the use of the characteristic polynomial for computing eigenvalues, especially those of symmetric matrices. All the same the author, after careful study and numerical experiments, asserts that computation of the roots of fairly high-order polynomials (by Bairstow or Muller's method) leads to meaningful results if sufficient care is taken.

The author's remarks can be summarized as follows. (1) The loss of accuracy in Newton's or similar methods can be compensated by computing with multiple precision. This may seem a trivial remark but insofar as those methods are very fast anyhow and since multiple precision is used only in the vicinity of a root, the time taken to compute a root is still very short compared with the time taken to prepare the equation (see however the remarks in (3) below). (2) If the roots are computed in order of increasing absolute value, then "dividing off" the computed roots and continuing with the reduced polynomial does not lead to a significant disturbance of the remaining roots. (3) Because of the sensibility of the roots to changes of the coefficients (e.g., the computation of the characteristic equation of a matrix) their evaluation must be done with multiple accuracy too. Since this indeed may be a lengthy computation, it is obviously the crux of the matter that only a part of this preparation must be done with high accuracy. In this respect Givens' method [J. Soc. Indust. Appl. Math. **6** (1958), 26–50; MR **19**, 1081] to transform a nonsymmetric matrix orthogonally into Hessenberg form is very advantageous, since this transformation, which is the bulk of the work, can be done with simple accuracy, whereas the computation of the characteristic polynomial of the Hessenberg matrix (which must be done with higher accuracy) is not very laborious. (4) As a warning against unconsidered transformations of equations before solving them the author refers to an equation

$$\prod_1^7 (x^2 + A_i x + B_i) - \varepsilon \prod_1^6 (x + C_i)^2 = 0,$$

where Muller's method using the polynomial in this form gave roots at least 10^{10} times more accurate than Bairstow's method applied to the polynomial in the usual form: $x^{14} + C_1 x^{13} + \cdots + C_{14} = 0$.

H. Rutishauser (Zürich)

322:

Collatz, Lothar. ★The numerical treatment of differential equations. 3d ed. Translated from a supplemented version of the 2d German edition by P. G. Williams. Die Grundlehren der mathematischen Wissenschaften, Bd. 60. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. xv + 568 pp. (1 plate) DM 98.00.

The first edition of this standard work was reviewed in MR **13**, 285 and the second in MR **16**, 962. The present translation (published as the third edition) was made from the second edition from which, however, it differs in detail in that throughout the book a large number of minor improvements, alterations and additions have been made, numerous further references to the literature included and new worked examples incorporated.

323:

Bertram, G. Verschärfung einer Fehlerabschätzung zum Ritz-Galerkinschen Verfahren von Kryloff für Randwertaufgaben. Numer. Math. **1** (1959), 135–141.

A very precise bound, depending on both n and x, for the maximum error in applying the Ritz-Galerkin method to the problem

$$y'' - q(x)y = r(x), \quad y(0) = y(1) = 0$$

is given, the approximating function being a sine polynomial of order n. *R. C. T. Smith* (Armidale)

324:

Chang, K. T. On the numerical solution of simultaneous linear differential equations. J. Soc. Indust. Appl. Math. **7** (1959), 468–472.

If one of the standard single-step methods is applied to a linear system $(dy/dx)+A(x)y+f(x)=0$, then successive approximation vectors y_n, y_{n+1} are linearly related, the matrices involved depending on n and the system in question. The author suggests that these matrices be calculated once and for all in cases where many solutions of the same system are required.

Walter Gautschi (Oak Ridge, Tenn.)

325:

Sauer, R. Application of numerical methods of the theory of characteristics to problems in gasdynamics. Symposium on the numerical treatment of partial differential equations with real characteristics: Proceedings of the Rome Symposium (28–29–30 January 1959) organized by the Provisional International Computation Centre, pp. 3–11. Libreria Eredi Virgilio Veschi, Rome, 1959. xii+158 pp.

Expository article. *J. H. Giese* (Aberdeen, Md.)

326:

Weinberger, H. F. Exact bounds for solutions of hyperbolic equations by finite difference methods. Symposium on the numerical treatment of partial differential equations with real characteristics: Proceedings of the Rome Symposium (28–29–30 January 1959) organized by the Provisional International Computation Centre, pp. 87–97. Libreria Eredi Virgilio Veschi, Rome, 1959. xii+158 pp.

Let u be a solution of the $(N+1)$-dimensional wave equation $\partial^2u/\partial x^{02}-c^2\Delta u=0$ for $x^0>0$, where for $x^0=0$, $u=f$ and $\partial u/\partial x^0=g$ on the cube $D:|x^i|<1$, $i=1,\cdots,N$, and $u=0$ on the boundary C of D for $x^0>0$. $f[g]$ is assumed to have square integrable derivatives of order 3 [2] on D and vanishes with its derivatives to order 2 [1] on C. On the cubic lattice $x^\alpha=m^\alpha h$, where m^α is an integer, $\alpha=0, 1, \cdots, N$, approximate u by the solution of the difference equation

$$E_0\psi-2\psi+E_0^{-1}\psi-c^2\sum_1^N(E_i\psi-2\psi+E_i^{-1}\psi)=0,$$

where the operator E_α [or E_α^{-1}] replaces x^α by $x^\alpha+$ [or $-$] h. Define initial data for ψ by

$$\psi(0,m^ih)=h^{-N}\int_{-h/2}^{h/2}f(m^ih+\xi^i)d\xi^1\cdots d\xi^N,$$

$$\psi(h,m^ih)-\psi(0,m^ih)=h^{1-N}\int_{-h/2}^{h/2}g(m^ih+\xi^i)d\xi^1\cdots d\xi^N,$$

and take $\psi(m^0h,m^ih)=0$ for $m^ih\in D$. Recall that u satisfies an "energy identity" which asserts that

$$\|\partial u/\partial x^0\|^2+c^2\sum_1^N\|\partial u/\partial x^i\|^2$$

is independent of x^0, where

$$\|\partial u/\partial x^0\|^2=\int_D(\partial u/\partial x^0)^2d\xi^1\cdots d\xi^N.$$

By exploiting the energy identity and an analogous "energy inequality" for ψ the author shows that

$$\|\psi(m^0h,m^ih)-u(m^0h,m^ih)\|^2/h^2$$

is uniformly bounded for all m^0, in terms of the known norms of $f[g]$ and their derivatives to order 3 [2], where

$$\|\psi-u\|^2=h^{-N}\sum_D(\psi(m^0h,m^ih)-u(m^0h,m^ih))^2.$$

J. H. Giese (Aberdeen, Md.)

327:

Loeb, Julien. La "déconvolution" en calcul numérique. Ann. Télécommun. **15** (1960), 84–91.

Résumé de l'auteur: "On transpose dans le domaine du calcul numérique l'équation intégrale

$$V(x)=\int_a^b g(u)z(x+u)du$$

où l'inconnue est la fonction z, $V(x)$ est une fonction enregistrée et g définit le brouillage dû à un appareil de mesure.—L'échantillonnage conduit à définir des matrices particulières à un nombre infini de termes, mais presque diagonales, que l'auteur propose d'appeler grilles. On en donne un mode d'expression au moyen de l'opérateur d'avancement E. On indique les règles de l'algèbre des grilles (la multiplication est commutative et l'inversion peut s'opérer suivant les règles de la division des polynômes).—On donne les conditions nécessaires et suffisantes pour que l'inversion soit possible et on indique un procédé pratique d'approximations successives pour le calcul de la grille inverse."

328:

Warmus, M. Nomographic functions. Rozprawy Mat. **16** (1959), 1–151.

The book deals with the problem of determining when a function $F(x, y, z)$ can be expressed in the form

$$F(x,y,z)=\begin{vmatrix}X_1(x)&X_2(x)&X_3(x)\\Y_1(y)&Y_2(y)&Y_3(y)\\Z_1(z)&Z_2(z)&Z_3(z)\end{vmatrix}.$$

The author gives computational schemes for the reduction of F to the required form and for finding all possible ways of so expressing F. *N. S. Mendelsohn* (Winnipeg, Man.)

329:

Wilson, Raymond H., Jr. Nomographic solutions for position relations between a close earth satellite and its observer. Amer. Math. Monthly **66** (1959), 393–401.

The apparent position of a satellite is specified by any three of the following four quantities: (1) the ground range, measured along the arc of a great circle from the observer to the subsatellite point, (2) the apparent azimuth, that is, the angle the ground range line makes with the North point of the compass, (3) the altitude, or the angle of elevation above the horizon, (4) the distance, or "the slant range". These quantities can be calculated from the satellite ephemeris, consisting of the latitude and the longitude of the subsatellite point at a given instant of time.

Applying spherical trigonometry to the astronomical triangle, the author develops the relevant formulae and

constructs nomograms based on these formulae. An example explains the use of the nomograms in furnishing quickly the approximate apparent position from the ephemeris data. *T. Leser* (Aberdeen, Md.)

330:

Chirol, Henri. Dispositifs pratiques facilitant l'analyse et la synthèse harmonique. Chiffres **2** (1959), 219–231. (English, German and Russian summaries)

This paper describes a simple mechanical and related graphical procedure to obtain approximate values at equidistant abscissas for the low-order harmonics of a numerically given function.

W. C. Rheinboldt (Syracuse, N.Y.)

331:

Centre National d'Études des Télécommunications. ★Tables numériques des fonctions associées de Legendre: Fonction associée de première espèce $P_n^m(\cos\theta)$. 2e fascicule. Collection Technique et Scientifique du C. N. E. T. Revue d'Optique, Paris, 1959. xii + 639 pp. (3 plates). 5600 francs.

These tables give Legendre and Associated Legendre Functions

$$P_n^m(\cos\theta) = C_{n,m}\frac{\sin^m\theta}{2^m}F(m-n, m+n+1; m+1; \sin^2\tfrac{1}{2}\theta)$$

with

$$C_{n,m} = \frac{(-1)^m}{m!}\frac{(n+m)!}{(n-m)!}$$

in which $F(a, b; c; x)$ is the hypergeometric function. An important feature is that n is not confined to integer values. In fact, values are given for $m=0, 1, 2$; $n=-0.5(0.1)+10$; $\theta=0°(1°)180°$. Thus several functions expressible in terms of elliptic integrals are represented; these occur whenever $2n$ is an odd integer.

In all cases, second and fourth central differences, possibly modified, are given to aid interpolation by use of the Everett-Laplace formula, except that when $\theta \geqq 135°$, the auxiliary function

$$T_n^m(\cos\theta) = \sin^m\theta[P_n^m(\cos\theta) + (-1)^m A_n \log_{10}\cot\tfrac{1}{2}\theta\, P_n^m\{\cos(180°-\theta)\}],$$
$$A_n = (2/\pi\log_{10}e)\sin n\pi$$

is given as well as $P_n^m(\cos\theta)$, and the differences are then given for $T_n^m(\cos\theta)$.

The precision of the tables is not simple to describe fully. The number of decimals varies from 8 or 9, when θ, m, n are small, to 5 when $m=2$, $n\to10$, $\theta\to180°$; this is for the interpolable tables. The number of decimals runs down to 2 for $P_n^m(\cos\theta)$ in this region.

A very brief introduction describes the calculation and use of the tables, and refers to L. Robin [*Fonctions sphériques de Legendre et fonctions sphéroïdales*, Tome II, Gauthier-Villars, Paris, 1958; MR **21** #734].

J. C. P. Miller (Madison, Wis.)

COMPUTING MACHINES

See also 315, 566.

332:

Sciama, Antoine. Adaptation de la méthode des moindres carrés aux machines à calculer électroniques, notamment pour la représentation mathématique de fonctions calculées ou empiriques. Chiffres **2** (1959), 157–180. (English, German and Russian summaries)

L'auteur expose les grandes lignes d'un programme réalisé pour une machine à calculer à tambour magnétique et déterminant des approximations au sens des moindres carrés avec éventuellement des équations de condition (points ou tangentes fixés). Le programme utilise des sous-programmes matriciels préexistant.

J. Kuntzmann (Grenoble)

333:

Phillips, Charles A. A common language to program computers for business problems. Computers and Automation **9** (1960), no. 1, 8–9, 23.

334:

Kappler, M. O. Computer programming for command control systems. Computers and Automation **9** (1960), no. 5, 8–14.

335:

Martens, Henrik H. Two notes on machine "learning". Information and Control **2** (1959), 364–379.

The article describes a program for the Bell Telephone Laboratories Leprechaun computer that learns to play a class of 3×3 board games including tic-tac-toe. The program works backwards in the move tree from final positions whose values are learned by experience. Heuristics involving the symmetries of the board and the inclusion of certain positions in others are also used.

The author also introduces the formal concept of L-automata which can learn, essentially by exhaustion, to win board games of a certain class.

J. McCarthy (Cambridge, Mass.)

MECHANICS OF PARTICLES AND SYSTEMS

See also A195, A204, 525.

336:

Nožička, František. Les principes fondamentaux de la mécanique et leur équivalence. Apl. Mat. **4** (1959), 243–277. (Czech. Russian and French summaries)

The motion of a system of N particles of m degrees of freedom may be represented as the motion of an abstract point in an m-dimensional variety immersed in the $3N$-dimensional (configurational) Euclidean space. Using the geometrical properties of regular varieties and further using d'Alembert's principle, the author gets the equations of motion in convariant form. These equations may be rewritten in the form usual in mechanics (Lagrange's equations of I and II kind, principles of Gauss and Hertz, Hamilton's canonical equations), whereby the equivalence of the mentioned principles is proved for the case in which the system is holonomic. From the mathematical point of view, the paper gives a unified and accomplished view of the fundamental equations of mechanics.

M. Brdička (Prague)

337:

Hermes, Hans. Zur Axiomatisierung der Mechanik. The axiomatic method. With special reference to geometry and physics. Proceedings of an International Symposium held at the Univ. of Calif., Berkeley, Dec. 26, 1957–Jan. 4, 1958 (edited by L. Henkin, P. Suppes and A. Tarski), pp. 282–290. Studies in Logic and the Foundations of Mathematics. North-Holland Publishing Co., Amsterdam, 1959. xi+488 pp. $12.00.

The various schemes leading to an axiomatisation of mechanics all employ the fundamental kinematical conceptions of position, velocity and acceleration, relative to a rigid reference frame, or a class of such frames mutually related by Galilei, or by Lorentz transformations. Most of these systems, however, in addition employ kinetical conceptions such as mass, impulse and force. The author, in the present paper, reviews briefly an earlier attempt of his, in which kinematical conceptions alone are used. {Others that might deserve mention here are Mach and Margenau, to mention just two writers on the subject.}

The fundamental consideration in favour of this approach is of course quasi-metaphysical: mass and force are "not immediately given". {Incidentally, velocity isn't either, let alone acceleration. This is a matter of degree.} The measurement of mass-values, for instance, presupposes the use of the spring-balance, or the laws of collision, or Newton's 3rd law of motion. Now kinematical axiomatisations have just this advantage, that they make explicit the fact that a fundamental law is presupposed for the definition (meaning) of mass and other similar quantities. Incidentally, different definitions then amount to different systems of mechanics. This means of course that one ties oneself down to a unitary meaning of mass, corresponding to a preference for some given defining law.

The author's axiom-system which is then formulated contains the interesting axiom that postulates the "existence of mass-relationships", the suggestion being that without this axiom (i.e., without the existence of binary collisions) the concept of mass in not defined.—To meet an obvious criticism he suggests that the axiom postulates only the possibility of inelastic collisions. We are not told, however, what kind of possibility is being referred to: logical, theoretical, or technical.

G. Buchdahl (Cambridge, England)

338:

Adams, Ernest W. The foundations of rigid body mechanics and the derivation of its laws from those of particle mechanics. The axiomatic method. With special reference to geometry and physics. Proceedings of an International Symposium held at the Univ. of Calif., Berkeley, Dec. 26, 1957–Jan. 4, 1958 (edited by L. Henkin, P. Suppes and A. Tarski), pp. 250–265. Studies in Logic and the Foundations of Mathematics. North-Holland Publishing Co., Amsterdam, 1959. xi+488 pp. $12.00.

The purpose of the paper is (1) to provide a system of axioms for classical rigid body mechanics, (2) to demonstrate their derivation from those of particle mechanics, and (3) to use this derivation to discuss the notion of "reduction" of theories in the natural sciences. The author suggests that there are certain difficulties in understanding reduction to imply no more than defining the primitive notions of *RBM* in terms of those of *PM*, and then deriving the laws of *RBM* from those of *PM*. The major difficulty for the formal scheme proposed is that according to it *RBM* is reducible to set theory, and that hence there seems no point in bothering with a reduction to *PM*. Following up an idea of Nagel concerning the meaning of reduction in the natural sciences, the author proposes a supplement to his formal system in order to show that "theories and the hypotheses represented by the 'definitions' involved in the reduction of one theory to another may be either true or false".

This is done by introducing the notion of "intended interpretation" or "model" (in Tarski's sense of model, not that used by physicists!). The reduction then involves the empirical hypothesis that there is a certain connection between the intended models of the secondary and primary theories, e.g., that every rigid body is composed of particles, and that there are certain physical relations between the masses, positions, etc., of the two systems. A satisfactory reduction of one theory to another hence must satisfy not only the previously mentioned formal criteria, must not only be formally reducible, but it must also be the case that the primary system's intended models satisfy its axioms. A "quasi-formal" account is then given of these relations.

G. Buchdahl (Cambridge, England)

339:

Thiry, Y. Remarques sur les équations canoniques de la mécanique. Ann. Scuola Norm. Sup. Pisa. (3) **13** (1959), 205–221.

The author introduces a matrix notation for the Hamiltonian equations of a mechanical system. He applies matrix notation to a change of variables, and deduces the classical conditions in terms of Poisson or Lagrange brackets for the new variables to be canonical. He also studies the method of variation of parameters in matrix notation. *P. Franklin* (Cambridge, Mass.)

340:

DiMaggio, F. L.; Bleich, H. H. An application of a dynamic reciprocal theorem. J. Appl. Mech. **26** (1959), 678–679.

341:

Bressan, Aldo. Equazioni di Hamilton nello spazio di Hertz per un sistema materiale olonomo a vincoli variabili. Ann. Univ. Ferrara. Sez. VII. (N.S.) **7** (1957/1958), 83–90. (French summary)

The author derives the Hamilton equations of a point in a n-dimensional space of Hertz [cf. T. Boggio, Atti. Accad. Naz. Lincei. Rend. (6) **17** (1933), 452–460] which corresponds to a holonomic dynamical system with variable constraints. The constraints are supposed to be scleronomic, i.e., independent of time, while the active forces are conservative. *T. P. Anđelić* (Belgrade)

342:

Vogel, Théodore. Systèmes dynamiques héréditaires. Cahiers de Phys. **13** (1959), 319–328.

The paper discusses some mathematical aspects of the theory of dynamical systems which can be described in terms of the principle of superposition with an exponential "memory" function. An introduction stresses the impor-

tance of such a treatment for the theory of systems (e.g., biological ones) which show the phenomenon of aging.

B. Gross (Rio de Janeiro)

343:

Skalak, Richard; Yarymovych, M. I. Subharmonic oscillations of a pendulum. J. Appl. Mech. **27** (1960), 159–164.

Authors' summary: "Large amplitude oscillations of a simple pendulum whose support moves with a prescribed vertical oscillation are studied by an approximate method. Subharmonics of order 1/2, 1/4, 1/6, and 1/8 are discussed and the theoretical results are compared with experiments. The stability of the theoretical steady states is investigated by the method of Andronow and Witt."

344:

Göcke, Hermann. Ein Beitrag zur Berechnung rheolinearer Schwingungen. Wiss. Z. Hochsch. Schwermaschinenbau Magdeburg **2** (1958), 15–44.

Quasiharmonic or rheolinear oscillations are described by Hill's differential equation: $\ddot{y}+[\lambda+\gamma\varphi(t)]y=f(t)$, where $\varphi(t)$ is a periodic function. For $\varphi(t)=\cos\alpha t$ the differential equation is associated with the name of Mathieu.

The very elaborate paper starts out with an account of known solutions where $\varphi(t)$ is a piecewise linear function. The general solution of the differential equation

$$\sum_1^n (a_\nu+b_\nu t)x^{(\nu)}(t) = 0,$$

as given by Laplace, is used to solve Hill's differential equation for the case of a "triangular" function $\varphi(t)$, i.e., $\varphi(t)$ a periodic, continuous and piecewise linear function.

The solution is given in terms of confluent hypergeometric functions, which happen to be Hankel functions of orders 1/6 and 5/6. The stability problem for the solution is discussed in great detail and an Ince-Strutt-stability diagram is given.

A comparison with Meissner's solution for piecewise constant $\varphi(t)$ shows that the regions of instability are very much alike for the two solutions.

H. Oser (Washington, D.C.)

345:

Tong, Kin N. ★Theory of mechanical vibration. John Wiley & Sons, Inc., New York-London, 1960. xii + 348 pp. $9.75.

This book is intended by the author primarily as a text for a "theory-centered" course in mechanical vibrations for seniors or first year graduate students in engineering. The reviewer believes that the book will be both informative and challenging, particularly to the more able student.

The book consists of four more or less equal chapters. The first three deal respectively with systems of one, two, and several degrees of freedom; moreover, each is divided into two parts, one treating theory and principles, and the other methods and applications. In the final chapter vibrations of continuous media are discussed; here the distinction between theory and application is not made. Finally there is a brief appendix on elementary matrix theory. There is a collection of problems at the end of each chapter.

The emphasis throughout is on methods of general applicability and theoretical interest, rather than on problem solving. For instance, Rayleigh's principle is first introduced on page 10 and is then elaborated in each succeeding chapter. Likewise, the ideas of orthogonality, influence coefficients, and superposition are carefully discussed in the earlier chapters before full use is made of them in continuum problems.

On the other hand, while the author has undertaken to write on a relatively advanced mathematical level, he faces the fact that a sizeable portion of his audience has no more than a rudimentary knowledge of advanced calculus and differential equations. The result is that there are a good many compromises, in the form of references to specific theorems in standard mathematical texts, proofs which are omitted, or others which are included under somewhat restricted conditions. To the author's credit, theorems and their limitations are carefully stated even when the arguments cannot be given. In the long run, as more engineering students take more mathematics, the value of this book should be enhanced.

W. E. Boyce (Troy, N.Y.)

346:

Magiros, Demetrios G. On a problem of nonlinear mechanics. Information and Control **2** (1959), 297–309.

Existence and stability of subharmonics of order 1/2 of

$$\ddot{Q}+Q+\varepsilon(k\dot{Q}-c_1Q+c_2Q^2+c_3Q^3) = B\sin 2t$$

are studied following a previous paper of the author [Information and Control **1** (1958), 198–227; MR **20** #5325].

J. L. Massera (Montevideo)

347:

Kondurar', V. T. Particular solutions of the general problem on the translational-rotational motion of a spheroid attracted by a sphere. Astr. Ž. **36** (1959), 890–901. (Russian. English summary); translated as Soviet Astr. AJ **3**, 863–875 (1960).

The particular solution of the title has the satellite's center of gravity traveling with constant angular velocity in a circular orbit while the satellite's axis, about which the satellite rotates with constant angular velocity, rotates about the normal to the orbital plane with a constant velocity of precession.

R. G. Langebartel (Urbana, Ill.)

STATISTICAL THERMODYNAMICS AND MECHANICS

See also 462, 483, 580, 581, 588, 592.

348:

Burley, D. M. A lattice model of a classical hard sphere gas. Proc. Phys. Soc. **75** (1960), 262–274.

Author's summary: "From an examination of the proposed lattice model of a hard sphere gas, several interesting features are observed. It is found that for loose packed lattices it is necessary to consider two regions, one ordered and one disordered, so that the whole pressure-density curve may be described. The disordered region, for these lattices, produces virial coefficients which oscillate in sign every three or four terms, and above a certain density the phase becomes unstable. Treatment of

the ordered region is not altogether satisfactory but it appears that the phase disappears below a certain density. Consequently, there is a strong indication that a transition occurs for loose packed lattices, but the nature and position of the transition is still in doubt. For the close packed lattices, however, it is found that the gas has virial coefficients which are all positive and so seems to be able to condense into an ordered phase without any transition."

349:
Murakami, Terutosi. On the variation principle in the kinetic theory of dense gases. I. J. Phys. Soc. Japan **15** (1960), 60–69.

A variational principle, leading to the Boltzmann equation, is given for the case of a dense gas consisting of rigid-sphere molecules with a non-zero radius.

The connection with entropy production is discussed.

S. Simons (London)

350:
Desloge, Edward A.; Matthysse, Steven W. Collision term in the Boltzmann transport equation. Amer. J. Phys. **28** (1960), 1–11.

A discussion is given of the Boltzmann transport equation for the case of particles of mass m interacting by means of hard elastic collisions with moving scatterers of mass M; the scattering probability is obtained in terms of the corresponding probability for collisions with rigid elastic centres. The results are applied to the conductivity of a slightly ionised gas. *S. Simons* (London)

351:
Blatt, J. M. An alternative approach to the ergodic problem. Progr. Theoret. Phys. **22** (1959), 745–756.

The author argues that approach to thermal equilibrium should be derived from random interactions between the system and the outside world, rather than by the usual "coarse-graining". He illustrates the ideas by analyzing a one-dimensional ideal gas enclosed by massive walls with a random velocity. *S. Katz* (New York)

352:
Bearman, Richard J. Statistical mechanical theory of the diffusion coefficients in binary liquid solutions. J. Chem. Phys. **32** (1960), 1308–1313.

Author's summary: "The statistical mechanical theory of the diffusion coefficients in binary liquid solutions is developed from the viewpoint of the pair space linear relations. Equations for the composition dependence of the ratios of the friction and diffusion coefficients are developed. For solutions which are regular it is found that the ratio of the two self-diffusion coefficients is in the inverse ratio of the corresponding molar volumes. The assumptions of the theory are discussed critically."

353:
Sah, Priyamvada; Eisenschitz, R. Electrons in weak interaction with a chain of atoms. Proc. Phys. Soc. **75** (1960), 700–712.

Authors' summary: "The weak binding approximation to the electronic energy spectrum is applied to a one-dimensional model of a liquid. The energy is expanded in a perturbation series, every term depending upon the positions of all atoms. The absence of long distance order is taken into account by assigning to the coordinates of the atoms a probability distribution by means of which the statistical expectation of the energy levels is obtained. Hereby the separations between neighbouring atoms are assumed to be statistically independent. At those energies which would be forbidden in a periodic chain the density of states is reduced but would vanish, presumably, only in the limit of a very large number of terms in the series being taken into account."

354:
Mori, Hazime. Correlation function method for transport phenomena. Phys. Rev. (2) **115** (1959), 298–300.

This is an improvement of a theory of irreversible processes previously proposed by the author [Phys. Rev. (2) **112** (1958), 1829–1842; MR **20** #6823]. The calculations are still very abbreviated, but it seems to the reviewer that some of the approximations of the original theory have been eliminated, and the premises are now quite unobjectionable. The theory leads to explicit expressions for the macroscopic currents in terms of the departures from thermodynamic equilibrium, and would allow the calculation of coefficients of thermal and electrical conduction, etc., if one could determine the quantum mechanical density matrix appropriate to the macroscopic system considered. The reviewer would like to see a more detailed development of the theory. *H. S. Green* (Adelaide)

355:
Kučerov, R. Ya.; Rikenglaz, L. Ė. Slipping and temperature discontinuity at the boundary of a gas mixture. Ž. Éksper. Teoret. Fiz. **36** (1959), 1758–1761. (Russian); translated as Soviet Physics. JETP **9**, 1253–1255.

Authors' summary: "Boundary conditions have been obtained for the hydrodynamical transport equations for a gas mixture. The conditions take into account slipping and the temperature jump at the boundary with a solid surface."

356:
Boffi, Vinicio C. On the slowing down of neutrons in an homogeneous infinite medium. Ann. Physics **9** (1960), 435–474.

The slowing down of neutrons in an infinite homogenous medium, if this medium consists of one element only, is governed by

$$(1)\quad N(z) = q\int_{z-1}^{z} \exp[(z-z_0)\log\alpha]h(z_0)\{N(z_0)+C'\delta(z_0)\}dz_0.$$

Most of the paper is devoted to the case where $h(z)=$ const. First a number of known results [Placzek, Phys. Rev. (2) **69** (1946), 423–438; MR **8**, 30; Marshak, Rev. Mod. Phys. **19** (1947), 185–238; MR **9**, 190] are rederived, in particular the asymptotic solution

$$(2)\qquad N(z) \sim C\exp[(1-\nu)z\log\alpha] \qquad \text{(large } z\text{)}.$$

The transcendental equation for ν in (2) is re-examined and some modifications are introduced into the methods of its solution. The author produces also an approximation

to $N(z)$ which agrees with the correct solution in the limits of $z \to 0$ and $z \to \infty$. An attempt is made to extend the same method to some cases where $h(z) \neq$ const.

The mixtures are treated by replacing them by a fictitious single element of a suitable mass number. This makes the results for mixtures rather unreliable and the entire treatment inapplicable when the cross-section of different constituent elements follow different laws of variation with energy. The proper treatment of mixtures had been discussed by Marshak [op. cit.].

B. Davison (Toronto, Ont.)

357:

Case, K. M. Elementary solutions of the transport equation and their applications. Ann. Physics **9** (1960), 1–23.

The paper is concerned with the neutron transport theory equation in one-velocity theory isotropic scattering case

$$\mu \partial\psi/\partial x + \psi = (c/2)\int_{-1}^{1} \psi(x, \mu')d\mu'. \tag{1}$$

If x dependence is eliminated by putting

$$\psi = \exp(-x/\nu)\varphi_\nu(\mu)$$

the equation becomes

$$(1-\mu/\nu)\varphi_\nu(\mu) = (c/2)\int_{-1}^{1} \varphi_\nu(\mu')d\mu'.$$

It has been known that, apart from regular solutions, (1) possesses singular solutions

$$\varphi_\nu(\mu) = (c/2)P[\nu/(\nu-\mu)] + \lambda(\nu)\delta(\mu-\nu), \tag{2}$$

where P indicates that on integration the principal value is to be taken, and δ is Dirac's delta function.

The author has now shown that the singular solutions (2), together with the regular solutions of (1), form a complete orthogonal set for functions in the interval $(-1, 1)$. A suitable sub-set of the above set is shown to be complete in any partial range $(-1 \leq \alpha < \beta \leq 1)$. The above results are used to rederive in a simpler manner the solutions of the Milne problem, the albedo problem, etc. Other applications are indicated. *B. Davison* (Toronto, Ont.)

358:

Wimmel, Hermann K. Asymptotische Entwicklung der Zustandssumme des starren symmetrischen Kreisels. Z. Naturf. **14a** (1959), 738–749.

Für die Theorie des dielektrischen Verhaltens polarer Gase im Mikrowellengebiet spielt die Größe der Zustandssumme eine entscheidende Rolle. Für den Fall eines Moleküls, das einen starren symmetrischen Kreisel bildet, wurde die Zustandssume bereits von Viney berechnet. Sie ergibt sich als Doppelsumme über die entsprechenden Quantenzahlen, wobei die statistischen Gewichte der verschiedenen Rotationszustände mit berücksichtigt werden. Für bestimmte Punktsymmetrie-Gruppen gehen nur die Mittelwerte der statistischen Gewichte ein, so daß vereinfachte asymptotische Entwicklungen möglich werden, die von der Molekülsymmetrie und vom Kernspin unabhängig sind. Die Entwicklungen gelten für abgeplattete und für verlängerte Kreisel und gestatten eine gute Näherungsberechnung der Zustandssumme in einem weiten Temperaturbereich. *K. Magnus* (Stuttgart)

359:

Reiss, Howard; Frisch, H. L.; Helfand, E.; Lebowitz, J. L. Aspects of the statistical thermodynamics of real fluids. J. Chem. Phys. **32** (1960), 119–124.

This is an extension of earlier work by Reiss, Frisch and Lebowitz [same J. **31** (1959), 369–380; MR **21** #3956], but introduces additional ideas borrowed from the 'hole' model of the liquid state. The interest of this paper is mainly from the point of view of its applications to the thermodynamics of solutions, to the surface tension of various substances, and to heats of vaporization. Where comparison with experiment is possible, the numerical agreement is surprisingly good. *H. S. Green* (Adelaide)

ELASTICITY, PLASTICITY

See also 345, 464.

360:

Shen, Mok-Kong. Eine Erweiterung des Maxwellschen Vertauschungssatzes. Ing.-Arch. **27** (1960), 350–351.

361:

Boley, B. A. On a dynamical Saint Venant principle. J. Appl. Mech. **27** (1960), 74–78.

362:

Eganyan, V. V. The plane problem of the theory of elasticity for a semicircle. Izv. Akad. Nauk Armyan. SSR. Ser. Fiz.-Mat. Nauk **11** (1958), no. 6, 47–59. (Russian. Armenian summary)

The load on the semicircle is distributed arbitrarily along the rectilinear and semicircular edges. The author uses bi-polar coordinates, the poles coinciding with the intersections of the semicircle with the diameter. The general formulas for stresses for any shape in bi-polar coordinates are taken from E. G. Coker and L. N. G. Filon, *A treatise on photo-elasticity* [Cambridge Univ. Press, Cambridge, 1931]. The resulting solutions are expressed in terms of Fourier integrals.

In the second part of the paper the author applies his general solution to several special cases of loading.

T. Leser (Aberdeen, Md.)

363:

Vorovič, I. I.; Kosmodamianskiĭ, A. S. Elastic equilibrium of an isotropic plate weakened by a series of uniform curvilinear holes. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 4, 69–76. (Russian)

364:

Watanabe, Yoshio. Stresses in a heavy cylinder resting on another so that their axes are parallel. Bull. Univ. Osaka Prefecture Ser. A **7** (1959), 1–10.

Author's summary: "In this paper, the state of stress in the upper cylinder due to its own weight is analysed as a two-dimensional elastic system, when a cylinder rests just upon the other one, the axis of which is parallel to that of the former."

365:
Ling, Chih-Bing. On invariant perforation in an infinite strip. J. Appl. Mech. **26** (1959), 422–431.

Invariant perforation of a strip is characterized by equal properties of all holes. A group of five cases of perforation by a finite number of holes and a group of nine cases of perforation by an infinite number of holes are considered. In the present paper the case of perforation by two rows of equal holes, symmetrically staggered along the strip, is investigated in detail. A class of special harmonic functions is constructed for this purpose, derived from Weierstrass' sigma function. In view of the invariant property of the perforation it is sufficient to satisfy the boundary conditions along the edge of the strip and along the edge of one hole. A numerical example illustrates an application of the theory. *W. T. Koiter* (Delft)

366:
Goldberg, M. A.; Sadowsky, Michael. Stresses in an ellipsoidal rotor in a centrifugal force field. J. Appl. Mech. **26** (1959), 549–552.

Authors' summary: "This paper contains an exact solution for the stress distribution in an elastic ellipsoid rotating about a free axis. The surface of the body is free from boundary stresses. The stresses are expressed with the use of 12 coefficients which have been tabulated for 209 ellipsoids of different shapes. General diagrams give the maximum values of stresses as functions of two shape ratios."

367:
Aleksandryan, R. A.; Arutyunyan, N. H.; Manukyan, M. M. Reaction problem of the bending of a prismatic rod. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 73–81. (Russian)

368:
Bloh, V. I. Torsion of elastic parabolic prisms. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 2, 166–169. (Russian)

369:
Di Prima, R. C. Coupled torsional and longitudinal vibrations of a thin bar. J. Appl. Mech. **26** (1959), 510–512.

Analysis is made of the coupling of torsional and longitudinal motions in a pretwisted unprestressed thin bar by a beam type theory. Computations are made for rectangular cross section bars of depth-to-width ratios of 1/10, 1/15 and 1/20 and for a Poisson ratio of 1/3. Comparisons are made of the coupled and uncoupled natural frequencies and only the torsional frequencies are found to be affected by the pretwist.
M. G. Scherberg (Dayton, Ohio)

370:
Uzdalev, A. I. Elastic equilibrium of a rod with cylindrical anisotropy under the influence of a load uniformly distributed lengthwise. Inžen. Sb. **26** (1958), 148–160. (Russian)

A prismatic bar of arbitrary cross-section and of finite length is considered. The axis of the cylindrical anisotropy of the material is parallel to the geometrical axis of the rod. The load acts on the periphery of the rod perpendicular to its axis and is uniformly distributed lengthwise. Author introduces three stress-functions and gives certain numerical results. *Z. Kączkowski* (Warsaw)

371:
Chakravorti, Amitava. A note on the position of the centre of flexure of a beam of isotropic material having a section bounded by a parabola and a straight line. Z. Angew. Math. Phys. **10** (1959), 333–338. (German summary)

372:
Stepanov, R. D. On some quadratures of beam fundamental functions. Inžen. Sb. **26** (1958), 94–100. (Russian)

373:
Keshava Rao, M. N. Bending of thin rectangular plates with edge beams. J. Appl. Mech. **26** (1959), 686–688.

374:
Fletcher, H. J. Bending of isosceles right triangular plates. J. Appl. Mech. **26** (1959), 625–628.

Two methods are presented to solve the title problem under arbitrary transverse loads. It can be treated as the case of a concentrated diagonal load on a square plate and, also, as the case of an antisymmetric load on a square plate. For clamped hypotenuse and simply supported legs, the numerical solutions obtained by the two methods have been shown to compare well. *S. C. Das* (Madras)

375:
Kurdin, N. S. Bending of a rectangular plate under local static loading with various boundary conditions. Inžen. Sb. **26** (1958), 199–204. (Russian)

376:
Popov, G. Ya. Deflection of a semi-infinite plate with combined elastic foundation. Dokl. Akad. Nauk SSSR **126** (1959), 534–537 (Russian); translated as Soviet Physics. Dokl. **4**, 692–696.

Author's summary: "In this paper, the exact solution is given to the problem of the deflection of a semi-infinite plate on an elastic foundation which is in the form of an elastic half-space (with constants E_0, ν_0), covered by a system of individual springs with stiffness k. A particular case of the problem is the problem of the deflection of a semi-infinite plate on an elastic half-space ($k=0$)."
R. C. T. Smith (Armidale)

377:
Baltrukonis, J. H. Ellipsoidal shells subjected to lateral hydrostatic loading. Proceedings of the Third U.S. National Congress of Applied Mechanics, Brown

University, Providence, R.I., June 11–14, 1958, pp. 247–252. American Society of Mechanical Engineers, New York, 1958. xxvii+864 pp. $20.00.

A linear membrane solution is obtained for the problem of a semi-ellipsoidal shell of revolution subjected to internal pressure varying linearly in a direction normal to the axis of revolution. This problem is encountered in the stress calculation of an ellipsoidal head of a fluid-filled, horizontal, cylindrical tank. Although it is generally known that the introduction of stress functions usually simplifies the solution of membrane problems of this type, no use of this fact is made by the author, and hence the solution is obtained after rather lengthy but straightforward calculations. A result of practical significance is the observation that circumferential compressive stresses exist in substantial portions of the shell introducing the possibility of buckling.

H. J. Weinitschke (Los Angeles, Calif.)

378:

Reissner, E. On the solution of a class of problems in membrane theory of thin shells. J. Mech. Phys. Solids **7** (1959), 242–246.

In the linear membrane theory of thin shells of revolution the use of Nemenyi's stress function leads to relatively simple solutions for many interesting special problems, as observed by Nemenyi and Truesdell [Proc. Nat. Acad. Sci. U.S.A. **29** (1943), 159–162; MR **5**, 84]. The author notes that an explicit solution (in terms of indefinite integrals) for arbitrary shells of revolution is possible when the stress function is of the form $U=f(z)\sin\theta$ ($r, \theta, z=$ cylindrical coordinates). He then applies his formulas to two special cases: (i) combined flexure and pure bending due to couples applied at the ends of a shell; (ii) a hydrostatically loaded ellipsoidal shell, for which an earlier solution has been obtained by Baltrukonis [see preceding review]. *H. J. Weinitschke* (Los Angeles, Calif.)

379:

Remizova, N. I. Calculation of cylindrical shells for strength by the integral equations method. Akad. Nauk Ukrain. RSR. Prikl. Meh. **4** (1958), 277–284. (Ukrainian. Russian and English summaries)

It is well known that the calculation of stresses in an arbitrary cylindrical shell is difficult when the rigorous, exact theory is combined with some special boundary conditions. By reducing the problem to a system of integral equations many of the difficulties can be removed. The author applies such a system of integral equations to an open cylindrical shell with a rectangular projection. She solves an example, shows that the solution is applicable only in a certain range of the shell's dimensions and compares her solution with that of V. Z. Vlasov [*General theory of shells and its applications in technology*, Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow-Leningrad, 1949; MR **11**, 627]. The agreement is very close.

T. Leser (Aberdeen, Md.)

380:

Èstrin, M. I. Calculation of a cylindrical shell clamped along an oblique contour. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 2, 151–155. (Russian)

381:

Csonka, P. Über doppelt gekrümmte Schalen. Acta Tech. Acad. Sci. Hungar. **26** (1959), 87–101. (English, French and Russian summaries)

Es werden doppelt gekrümmte kugelartige Schalen über viereckigen Grundrissen behandelt. Der Verfasser geht von Schalenformen aus, bei denen in den Formeln der Mittelfläche ein oder mehrere unbestimmte Parameter vorkommen. Zu diesen Schalenformen wählt er als Spannungsfunktion den Randbedingungen vollkommen entsprechende Funktionen, in denen ebenfalls ein oder mehrere unbestimmte Parameter auftreten. Den in den Formeln der Mittelfläche und der Spannungsfunktion vorkommenden unbestimmten Parametern werden nachträglich solche Werte zugeordnet, daß die durch die Differentialgleichung der Aufgabe bestimmten Werte der Lasten in einigen angenommenen Werten der Mittelfläche mit der tatsächlichen Belastung übereinstimmen. Das Verfahren wird an mehreren Beispielen erläutert. Es wird nachgewiesen, daß der durch das Näherungsverfahren entstandene Fehler unbedeutend ist.

W. Zerna (Hannover)

382:

Fradlin, B. N.; Šahnovskiĭ, S. M. Functional equations of equilibrium of sloping shells. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 144–149. (Russian)

383:

Rudis, M. A. Calculation of symmetric deformations of a conical and a spherical shell of constant thickness. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 2, 147–150. (Russian)

384:

Schmidt, R.; Wempner, G. A. The nonlinear conical spring. J. Appl. Mech. **26** (1959), 681–682.

385:

Burmistrov, E. F. Non-linear transverse oscillations of orthotropic shells of revolution. Inžen. Sb. **26** (1958), 5–20. (Russian)

Free and forced vibrations of spherical and conical shallow shells and plates of cylindrical anisotropy are considered. The Bubnov-Galerkin method is used. Author obtains the eigenfrequencies of free vibrations of a folded plate with four various boundary conditions.

Z. Kaczkowski (Warsaw)

386:

Rabinovič, B. I. On the equations of elastic vibrations of thin-walled bars filled with fluid in the presence of a free surface. Izv. Akad. Nauk. SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 4, 63–68. (Russian)

387:

Ellington, J. P.; McCallion, H. The free vibrations of grillages. J. Appl. Mech. **26** (1959), 603–607.

Authors' summary: "Frequency equations and modal shapes are derived for the transverse vibrations of beam

grillages consisting of two sets of equally spaced uniform beams, the mass of the beams and associated plating being taken as lumped at the nodes."

388:
Bodine, R. Y. The fundamental frequency of a thin, flat, circular plate simply supported along a circle of arbitrary radius. J. Appl. Mech. **26** (1959), 666–668.

389:
Kito, Fumiki. On vibration of an elastic bar which is immersed in a cylindrical water tank. Proc. Fac. Engrg. Keio Univ. **10** (1957), 49–62.

390:
Kito, Fumiki. On vibration of an elastic bar immersed in the water region lying between the two parallel plane walls. Proc. Fac. Engrg. Keio Univ. **10** (1957), 77–87.

391:
Yu, Yi-Yuan. Simple thickness-shear modes of vibration of infinite sandwich plates. J. Appl. Mech. **26** (1959), 679–681.

392:
Romanovskiĭ, Yu. M.; Strelkov, S. P. On the effect of atmospheric turbulence on an aeroplane with elastic wings for various flight velocities. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 4, 3–10. (Russian)

393:
Hamada, Minoru. Compressive or shearing buckling load and fundamental frequency of a rhomboidal plate with all edges clamped. Bull. JSME **2** (1959), 520–526.

394:
Machnig, Oskar. Über Stabilitätsprobleme von torusförmigen Schalen. Wiss. Z. Hochsch. Verkehrswes. Dresden **4** (1956), no. 2/3, 179–204. (Russian, English and French summaries) (1 plate)

Die Arbeit beginnt mit einer Zusammenstellung der Voraussetzungen für Stabilitätsuntersuchungen für Schalen. Es werden dann mit Hilfe des Tensorkalküls die Koeffizienten der ersten und zweiten Fundamentalform, sowie die Christoffelsymbole für die Torusfläche angegeben. Für die Lösung des Eigenwertverhaltens wird ein der numerischen Rechnung zugängliches Lösungsverfahren entwickelt, das die Aufstellung einer allgemeinen Beulformel für den Druck der geschlossenen Torusschale in Abhängigkeit von den geometrischen Kenngrößen gestattet. Der kleinste kritische Druck kann einem Diagramm entnommen werden. Weiterhin wird für Teilstücke von Torusschalen der Rechnungsgang zur Ermittlung des kritischen Druckes dargestellt. Damit gewinnt das Verfahren Bedeutung für die Untersuchung der Stabilität doppelt gekrümmter Schalenstreifen.

W. Zerna (Hannover)

395:
Brinis, Elisa. Sulla propagazione ondosa in un mezzo isotropo per strati. Ist. Lombardo Accad. Sci. Lett. Rend. A **93** (1959), 639–652.

J. L. Synge [J. Math. Phys. **35** (1957), 323–334; MR **19**, 83] has studied the propagation of plane harmonic waves in a medium with transverse isotropy (isotropy in plane layers). In the present paper the results are extended to the case in which the layers are not necessarily plane. The chief tool employed is the propagation ellipsoid, the axes of which are in length inversely proportional to the magnitudes of the three wave velocities. It is found that of the three directions in which discontinuities are propagated, in general one is transverse and the other two are neither transverse nor longitudinal; for special cases, however, the last two become aligned.

The splitting of the wave surface into a spheroid and a sheet that is a quartic surface, discussed by Synge for plane waves, is shown to persist for waves propagated from an epicenter when the layers are plane, homogeneous, and equal to one another. *R. N. Goss* (San Diego, Calif.)

396:
Sveklo, V. A. Sc!ution of a dynamic problem with mixed boundary conditions in the theory of elasticity for a half-plane. Prikl. Mat. Meh. **23** (1959), 266–273 (Russian); translated as J. Appl. Math. Mech. **23**, 383–393.

Author's introduction: "This article considers a problem describing the dynamic response of an elastic half-plane to the impact of a system of stamps in the absence of friction and cohesion."

C. E. Pearson (Cambridge, Mass.)

397:
White, J. E. Use of reciprocity theorem for computation of low-frequency radiation patterns. Geophysics **25** (1960), 613–624.

Author's summary: "Starting with a simple word statement of the reciprocity which exists between forces and displacements in a general elastic solid, it is shown that low-frequency radiation of shear and compressional waves from relatively complex sources can be obtained by solving relatively simple problems in static elasticity. Illustrative examples include radiation from radial and tangential pairs of forces acting on the wall of a cylinder, pressure in a finite cylinder, and a pair of radial forces in a hole in a plate. For the last case, measurements in a plexiglas plate compare favorably with computations."

398:
Borodačev, N. M. On the construction of general solution of dynamic problem of elasticity for half-space in case of axisymmetry. Bul. Inst. Politehn. Iaşi (N.S.) **4** (8) (1958), no. 3–4, 85–88. (Russian. English and Romanian summaries)

Author's summary: "In this paper a general solution is presented for the oscillations of an elastic isotropic half space $z \geq 0$, on condition that at $t \leq 0$ the half-space was motionless and at the moment when $t=0$ some source of an elastic perturbation independent of φ starts functioning on plane boundary of the half-space (using cylindrical coordinates (r, φ, z), the axis of z is drawn into the body).

"This solution is obtained with the help of Laplace and Henkel transforms.

"The application of obtained general solution to solving the particular boundary problems is also considered in this paper."

399:

Spencer, T. W. The method of generalized reflection and transmission coefficients. Geophysics **25** (1960), 625–641.

Author's summary: "The objective of this work is to provide a method for predicting the surface response of a stratified half space to the radiation from a localized source when neither the assumptions of the plane wave theory nor the assumptions of the normal mode theory are valid. The earth model consists of a finite number of perfectly elastic, homogeneous, isotropic layers separated by interfaces which are plane and parallel to one another.

"The method leads to an infinite series for the Laplace transform of the response function (displacement, velocity, stress, etc.) in a multi-interface system. Each term in the series describes all the energy which traverses a particular generalized ray path between the source and the receiver. The specification of the mode of propagation across each stratum (either as an irrotational wave or as an equivoluminal wave) and of the sequence in which the strata are traversed serve to define a generalized ray path. A prescription is given for constructing the integral representation for the disturbance which has traversed such a path directly from the integral representation for the source radiation. The method therefore obviates the necessity for solving a tedious boundary value problem. The time function associated with each term can be obtained by using Cagniard's method."

400:

Adkins, J. E. Symmetry relations for orthotropic and transversely isotropic materials. Arch. Rational Mech. Anal. **4**, 193–213 (1960).

The author solves the problem of determining the general form of a 3×3 symmetric matrix polynomial function of n 3×3 symmetric matrices which is invariant under the group of rotations about a fixed line. He uses the result to formulate a nonlinear viscoelasticity theory for transversely isotropic materials. He briefly indicates how to formulate a corresponding theory of orthotropic media.

J. L. Ericksen (Baltimore, Md.)

401:

Segawa, Wataru. Maxwell's formula for three-dimensional and large deformation. J. Phys. Soc. Japan **15** (1960), 339–344.

The author proposes a viscoelasticity theory which is intended to be a nonlinear version of the commonly used linear Maxwell theory and uses it to analyze simple tension. The theory involves actual displacement, a fictitious displacement and stress, or twelve unknowns. His equations (3.4), (4.1) and the usual equations of equilibrium provide fifteen equations to determine these. Considering the forms of the equations, it seems to the reviewer very likely that the system is overdeterminate. If so, solutions may be quite scarce. The author does not explain away or even acknowledge this difficulty.

J. L. Ericksen (Baltimore, Md.)

402:

Druyanov, B. A. Limit analysis of a plastic nonhomogeneous wedge. Dokl. Akad. Nauk SSSR **127** (1959), 990–992 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 916–918.

The paper is concerned with the limiting state of equilibrium of an infinite rigid, perfectly plastic wedge that is subjected to a uniform pressure on one of its sides. It is assumed that the yield stress of the material at the typical point of the wedge depends exponentially on the polar angle of this point.

W. Prager (Providence, R.I.)

403:

Zadoyan, M. A. Creep of compound prismatic rods under constrained torsion. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 82–88. (Russian)

404:

Baker, W. E. The elastic-plastic response of thin spherical shells to internal blast loading. J. Appl. Mech. **27** (1960), 139–144.

The author, jointly with F. J. Allen, has shown in Proc. 3rd U.S. National Congress Appl. Mech., pp. 79–87, Amer. Soc. Mech. Engrs.; New York, 1958 [MR **21** #1038] that the elastic response of a spherical shell of any thickness to a spherically symmetric transient pressure pulse on its inner surface can be adequately approximated by an appropriate thin-shell equation of motion. In this paper the author assumes that this can be extended to an elasto-plastic response as well. During plastic deformations both perfectly plastic and strain-hardening materials are considered. The author solves explicitly the linear equations of motion which result from assuming that thinning and increase in radius of the shell occuring during plastic deformation are negligible. The non-linear case results from accounting for thinning and from assuming that the shell material is incompressible. The latter case had to be solved numerically on the Aberdeen Proving Ground high speed digital computer "ORDVAC". The numerical solution agrees well with experiments and shows also that the thinning cannot be neglected.

T. Leser (Aberdeen, Md.)

405:

Kaliski, Sylwester; Osiecki, Jan. Unloading wave for a body with rigid unloading characteristic. Proc. Vibration Problems **1959**, no. 1, 49–66. (Polish and Russian summaries)

The propagation of plane waves in semi-infinite bars is considered assuming that the $\sigma\sim\varepsilon$ relation in the loading region is a convex curve towards the σ axis and in the unloading region is a vertically straight segment (rigid unloading). In examples, the $\sigma\sim\varepsilon$ curve in the loading region is considered a straight line too. The particle velocity and the variation of the stress in the unloading region are calculated supposing that the form of unloading waves (that is, the boundary between the loading and unloading region in the x, t plane) is known. In examples different increasing and decreasing laws of the stress at the end of the bar are considered.

N. Cristescu (Bucharest)

406:

Kaliski, Sylwester; Osiecki, Jan. The problem of reflection by a rigid or elastic wall of an unloading wave in a body with rigid unloading characteristic. Proc. Vibration Problems **1959**, no. 1, 83–99. (Polish and Russian summaries)

Using the results obtained in another paper [see the preceding review] the reflection of plane waves which propagate in a finite bar is considered. The $\sigma \sim \varepsilon$ relation is supposed to be composed by a straight line in the loading region and a vertically straight line in the unloading region (an elastic-rigid material). At the end of the bar the load is sudden and then decreases monotonically. The other bar-end is supported by an elastic or a rigid wall. *N. Cristescu* (Bucharest)

407:

Il'yušin, A. A. Some questions in the theory of plastic flow. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk **1958**, no. 2, 64–86. (Russian)

The author applies the theory of plastic flow to the analysis of the processing of metals by pressure. The state of stress of a plastically deformed body and its "shear resistance" depend on the temperature, velocity of deformation and magnitude of deformation. The author gives examples of approximate analytical expressions for stresses as functions of all the arguments indicating that in the heat processing stresses could be regarded as functions of temperature only. The analysis of the state of stress of a plastically deformed body requires a solution of a system of partial differential equations consisting of: (1) the equation of motion of the deformed medium in Eulerian coordinates, (2) the equation obtained from the condition of noncompressibility of the material, (3) the equation obtained from the condition of proportionality of the stress deviator to the corresponding component of the deformation velocity, (4) the heat balance equation. The author discusses the possible boundary conditions and shows approximate methods of solution in certain cases. For example, when the stress depends chiefly on the rate of change of the velocity of deformation the theory of small plastic deformations is applicable, and when the stresses are approximately constant in space the analogy of flow of a granulated medium is applicable. Finally the author considers a problem of settling of a metallic stratum in plane parallel layers, assuming that the variation of stresses in the stratum is due to the changes in temperature.
T. Leser (Aberdeen, Md.)

408:

Hoff, N. J.; Jahsman, W. E.; Nachbar, W. A study of creep collapse of a long circular cylindrical shell under uniform external pressure. J. Aero/Space Sci. **26** (1959), 663–669.

A long, thin-walled, almost-circular cylindrical shell is subjected to an external pressure which is constant in both time and space. The pressure is small compared to the plastic collapse load or the elastic buckling load so that failure of the shell if it occurs will be due to creep.

The following simplifying assumptions are made in the analysis: the shell is of idealized sandwich construction; the shell is sufficiently long that end conditions and axial bending moments may be neglected; the initial and all subsequent deviations from circularity are two-lobed so that the radius is of the form $R = B + \rho X \cos 2\phi$, where ρ is the shell wall radius of gyration and $X = X(t)$ a weighted shape factor of the section; elastic, plastic, and primary creep deformations are negligible, and secondary creep is governed by a power creep law.

The curvature rate at any time and place is determined as a function of external pressure and current shape and equated to the geometrical formula for curvature. Since an approximate shape was initially chosen, the agreement is not exact but it agrees at the maximum, minimum, and zero points. The time dependence of the shape factor is determined by solving the differential equation obtained when the curvature expressions are forced to further agree on the average over each octant. For the particular case of a cubic law the results are obtained in closed form.

It is found that the deformation amplitude will become infinite in a finite "critical" time, but that the deformation remains at its initial order of magnitude up to about 90% of this critical time. Further, if the critical time is computed as a function of the initial value X_0 of the shape factor, it is found to be relatively insensitive to small variations of X_0, particularly if X_0 is moderately large.
P. G. Hodge, Jr. (Chicago, Ill.)

409:

Mróz, Zenon. The load carrying capacity and minimum weight design of annular plates. Rozprawy Inż. **6** (1958), 603–626. (Polish. Russian and English summaries)

The paper consists of two parts. In the first part the author analyses the load carrying capacity of annular plates with arbitrary transverse radially-symmetric loads q with the following boundary conditions: (a) the outer edge or the inner edge is freely supported, (b) the outer or the inner edge is fixed, (c) both edges are freely supported. The author assumes the theory of plates of medium thickness, that is, the influence of the shearing and the normal stresses, perpendicular to the middle surface of the plate on the plasticity condition and on the state of deformation, is neglected. The material is assumed to conform to the "rigid-plastic" model and to satisfy the Coulomb-Tresca plasticity condition.

In the second part of the paper the author finds the optimum shape for annular plates of variable thickness. By the optimum shape the author means a shape which would require the least amount of material at a given load (the minimum weight design). The general analysis of this variational problem is followed by examples where the load q is assumed to be constant and various kinds of supports are considered. *T. Leser* (Aberdeen, Md.)

410:

Hwang, Chintsun. Incremental stress-strain law applied to work-hardening plastic materials. J. Appl. Mech. **26** (1959), 594–598.

Though long recognized as physically unsound, the so-called total stress-strain laws of plasticity have been widely used on account of their mathematical simplicity. To show that the inherent mathematical difficulties of the physically preferable incremental laws can be readily overcome by use of numerical integration on a digital computer, the author treats a simply supported and uniformly loaded circular plate that consists of a rigid, work-hardening material. The effects of successive small increases in the load intensity are investigated, and dia-

grams for the radial and circumferential bending moments and curvatures as well as the deflections are presented for a number of load intensities. {In view of recent and in the reviewer's opinion not well-supported arguments in favor of the use of total laws in the approximate analysis of plastic stresses and strains, it would be desirable to treat a wide variety of problems in this manner and compare the results with those obtained by the use of the corresponding total stress-strain laws.}

W. Prager (Providence, R.I.)

411:

Rennie, B. C. On the strength of sand. J. Austral. Math. Soc. **1** (1959/61), part 1, 71–79.

Assuming that an aggregate of identical, small, hard spheres arranged in the closest possible packing behaves in much the same way as a mass of dry sand, a theory is developed which describes the response of the aggregate to applied stresses and specifies conditions under which collapse takes place.

In Part 1 of the paper it is supposed that the idealized grains are perfectly smooth. The distance between the centres of any two grains is expressed in terms of the packing matrix and, by considering the transformation of this expression under a small strain, it is shown that for such a strain to be geometrically possible its components must satisfy six inequalities. These inequalities define a convex cone in 6-dimensional strain space, and it is supposed that collapse will take place when the virtual work performed by the applied stress acting through the strain components associated with a point inside or on this cone is positive. This criterion is then simplified, the final conclusion being that the frictionless aggregate will not collapse provided that the principal stresses are all compressive and the largest of them is less than twice the smallest. In Part 2 the model is generalized to include frictional forces at intergranular contacts, the analysis being valid, however, only when the coefficient of friction μ is sufficiently small. The stress matrix in the state of incipient collapse is evaluated under the assumption that friction is then limiting at all contacts, and it is deduced that the frictional model will remain rigid if the principal stresses are all compressive and the quotient of the largest and smallest is less than $2+2(\sqrt{6})\mu+$unknown higher powers of μ. Part 3 of the paper contains some remarks on the observed behaviour of natural sands, and a method of including the effects of complete or partial saturation in the foregoing analysis is outlined. The author does not attempt, however, to compare his results with the Coulomb yield criterion which is widely used in soil mechanics.

The various physical assumptions made in this paper are unlikely to recommend it to readers having a practical interest in soil mechanics, but the mathematical techniques introduced here form a useful contribution to the problem of predicting the bulk behaviour of sand from the properties of individual grains.

P. Chadwick (Sheffield)

412:

Fedorov, I. V. Some problems of the elastic-plastic distribution of stress in soils, connected with calculation of the foundations. Inžen. Sb. **26** (1958), 205–215. (Russian)

Stress field in an obtuse wedge under uniformly distributed load is analyzed with the assumption that the Coulomb yield criterion for cohesive soils is valid. Variation of stress field due to the variation of internal friction and cohesion is discussed.

No attempt is made to check the geometrical (kinematical) admissibility of the obtained solution and therefore no proof is given that the stress field corresponds to the exact solution.

A. Sawczuk (Warsaw)

413:

Coleman, Bernard D.; Noll, Walter. On the thermostatics of continuous media. Arch. Rational Mech. Anal. **4**, 97–128 (1959).

This is a clear, precise treatment of the thermostatics of continua, applicable to materials whose equilibrium behavior is adequately described by nonlinear thermoelasticity theory. Among other things, this includes elastic solids, perfect gases and viscoelastic materials.

The authors state: "We regard the main tasks of the science of thermostatics to be, first, the exploration of the consequences for the caloric equation of state of the existence of local states of thermal equilibrium and, second, the derivation of useful necessary and sufficient criteria for global states to be stable." This provides a succinct description of the paper.

J. L. Ericksen (Baltimore, Md.)

414:

Novackiĭ, Vitol'd [Nowacki, Witold]. Some three-dimensional problems of thermoelasticity. Prikl. Mat. Meh. **23** (1959), 456–467 (Russian); translated as J. Appl. Math. Mech. **23**, 651–665.

415:

Nowacki, Witold. Transient thermal stresses in viscoelastic bodies. I. Arch. Mech. Stos. **11** (1959), 649–669. (Polish and Russian summaries)

STRUCTURE OF MATTER

See also 565.

416:

Wannier, Gregory H. Wave functions and effective Hamiltonian for Bloch electrons in an electric field. Phys. Rev. (2) **117** (1960), 432–439.

The solution of the Schrödinger equation in a periodic potential field (crystal lattice) with a superposed homogeneous electric field presents a difficult question in solid state theory. If interpreted literally, it has no solution, and the problem is to re-formulate it in a manner which corresponds to the real physical situation. The author develops a recursive process for expressing the wave functions as a power series in the field strength, while preserving at the same time, so far as is possible, the concept of Brillouin zones. The theory is interpreted in a way which allows the use of an effective Hamiltonian operator for electrons in a given non-degenerate energy band. The treatment is formal, but the author hazards some comments on its convergence properties.

E. L. Hill (Minneapolis, Minn.)

417:

Nozières, Philippe. Effet de l'interaction de Coulomb sur les excitations élémentaires dans les solides. Ann. Physique **4** (1959), 865–903.

The theory of electron motions in solids, as it is usually presented, assumes that the cancellation of positive and negative electrical charge is so complete that each electron can be considered to move in an averaged field. It is recognized, however, that owing to the long range character of the Coulomb law of force the electrostatic forces can give rise to correlations of the motions of the electrons over large distances. These effects can be interpreted as plasma motions of the electrons. The author investigates the general theory of such oscillations, making use of the usual perturbation techniques. For an application of the theory see P. Nozières and D. Pines [Phys. Rev. (2) **113** (1959), 1254–1267; MR **21** #1058].

E. L. Hill (Minneapolis, Minn.)

418:

Gruber, Boris. Fundamental concepts of the theory of dislocations. Czechoslovak J. Phys. **9** (1959), 701–709. (Russian summary)

It is customary to visualize crystals as Bravais lattices, which are lattices invariant under a group of discrete translations and rotations. The author discusses geometrical problems arising when one attempts to compare such a model with a real crystal containing some imperfections. *J. L. Ericksen* (Baltimore, Md.)

419:

Loeb, A. L. A binary algebra describing crystal structures with closely-packed anions. Acta Cryst. **11** (1958), 469–476.

Author's summary: "A description of structures with close-packed anions is given in terms of simple, interpenetrating component lattice arrays. Each component array is denoted by a set of binary digits, from which the spatial relationship of these arrays can be derived. For ferrimagnetic rocksalt-like structures the magnetic dipole-dipole energy is minimized subject to exchange restraints, and it is shown that dipole patterns so obtained match those found by neutron-diffraction, and are not necessarily parallel and antiparallel to a single direction. Spinel and olivine structures are found to be ordered mixtures of some component arrays found in rocksalt structures and some found in sphalerite structures."

420:

Morris, I. L.; Loeb, A. L. A binary algebra describing crystal structures with closely packed anions. II. A common system of reference for cubic and hexagonal structures. Acta Cryst. **13** (1960), 434–443.

Authors' summary: "In the cubic crystals of the rocksalt, sphalerite, antifluorite, and spinel types and in the hexagonal crystals of the nickel arsenide, wurtzite, and olivine types, the cations are either octahedrally or tetrahedrally surrounded. Algorithms are presented here for locating the ions of these seven structures in the following sequence. First, cubically and hexagonally closely packed anion structures are compared, and geometric operators relating the two classes are defined. Second, the location of all possible cation sites is described algebraically, and finally the distribution of cations over these sites is described in terms of the parities of the coordinates of the sites. An interstitial type of crystal model from which the seven classes of crystals have been constructed is discussed."

421:

Löfgren, Tor. An assessment of the power of the non-negativity criterion as used in X-ray crystal structure determinations. Acta Cryst. **13** (1960), 429–433.

Author's summary: "It is shown why the requirement of reality coupled with that of non-negativity and certain symmetry of the electron-density function constitutes a weak criterion in space groups lacking translational symmetry elements, at least in so far as the requirements mentioned are exploited by any Harker-Kasper inequalities. In space groups with translational symmetry operations the situation is fundamentally different."

422:

Kimura, Katsumi; Kimura, Masao. Application of least-squares method to density-intensity calibration in electron diffraction studies. J. Chem. Phys. **32** (1960), 1398–1401.

Authors' summary: "A new method based on the principle of least squares is proposed for the calibration of the characteristics of photographic emulsion employed in electron diffraction studies. The calibration procedure consists of two steps. First, a function having three parameters was found which represents the relation between the optical density and exposure. Secondly, the most probable values of the parameters and their standard errors were determined in a straightforward way by the method least squares. An application of this method to the diffraction patterns of benzene is described. The standard errors of final relative intensities were evaluated over the entire observed range of optical densities."

FLUID MECHANICS, ACOUSTICS

See also 325, 483, 484, 513, 516, 618, 619.

423:

Robinson, Allan R. The symmetric state of a rotating fluid differentially heated in the horizontal. J. Fluid Mech. **6** (1959), 599–620.

From the author's summary: "The motion of a fluid inside a rotating annulus of square cross-section, whose dimensions are small compared with the distance from the axis of rotation, is considered. The rigid side walls are held at different constant temperatures, and the fluid motions that occur are strongly influenced by Coriolis accelerations. A detailed study is made of the azimuthally independent state, employing a boundary layer type analysis so that all the imposed boundary conditions are rigorously satisfied.

"An entirely geostrophic thermal wind is found to obtain over the main body of the fluid. The circulation in the plane of the annular cross-section is entirely confined within narrow boundary layers and consists of a superposition of three cellular motions: a cell occupying the

cross-section and two additional cells confined to the side-wall boundary layers. These motions are intimately related to the rotational constraint. The temperature distribution and its relation to the conductive and convective processes are determined."

M. H. Rogers (Bristol)

424:

Simasaki, Tamedi; Niu, Keishiro. On a method of successive approximations applied to the axisymmetric flows of an incompressible perfect fluid. J. Phys. Soc. Japan **14** (1959), 1810–1815.

A complex velocity potential F is defined in terms of the velocity potential and Stokes' stream function for a steady axisymmetric flow of incompressible inviscid fluid and an equation satisfied by F and its complex conjugate found. To solve this equation for a uniform flow past an axisymmetric body a general method of successive approximation is developed and applied to the flows past a sphere and a prolate spheroid. The approximate solutions so obtained in these cases are compared numerically with the known exact solutions. The method can be applied to the flow past any arbitrary axisymmetric body.

W. D. Collins (Newcastle-upon-Tyne)

425:

Ōba, R. Theory of flow through profiles with large camber and thickness arranged in cascade with small pitch chord ratio. Ing.-Arch. **27** (1959), 276–284.

426:

Drake, D. G. Quasi-steady derivatives for the subsonic flow past an oscillating aerofoil in a porous wind tunnel. Aero. Quart. **10** (1959), 211–229.

The paper extends some previous work by the same author [Aero. Quart. **8** (1957), 226–239; MR **19**, 703]. The derivatives for pitching motion about the mid-chord of the aerofoil are computed when quasi-steady conditions are assumed. As before, Fourier transform methods are employed to obtain solutions of the differential equation, satisfied by the velocity potential, which also satisfy the necessary boundary conditions. At the walls, the condition is taken to be "the normal velocity is proportional to the pressure drop across the wall". The conclusions drawn are that, for low frequency oscillations, the effect of the walls on l_a and n_a is zero for all Mach numbers, but the effect on l_2 and n_a is large for all values of Mach number and porosity.

G. N. Lance (Dorset)

427:

Ferguson, Harry; Guderley, Karl Gottfried. Extension of Evvard's method to oscillatory supersonic flow. Z. Flugwiss. **8** (1960), 76–86. (German and French summaries)

Authors' summary: "The computation of the pressure distribution on a thin wing oscillating in supersonic flow leads to a Volterra integral equation of the first kind with a singular kernel. Exact solutions in the form of an integral exist for steady supersonic flow with an angle of attack, and for oscillatory flows with only supersonic leading edges. On the basis of these exact solutions a hypothesis is made that transforms the original equation into a Volterra integral equation of the second kind with a fairly simple kernel. This equation is much more tractable than the original equation; for the known cases the kernel is identically equal to zero."

428:

Bessho, Masatoshi. On the theory of the wave resistance. J. Zosen Kiokai **105** (1959), 1–6. (Japanese. English summary)

Author's summary: "The author introduces the velocity potential of the displacement ship by making use of Green's function or the fundamental singularity. This method is more rational than the one we have had and deduces the result that singularities which are on the water line of the ship's surface are of negligible order, especially when the surface of the ship near this water line is vertical. Next, he obtains the integral equations of solving the problems by the aid of Green's function in the potential theory in cases of submerged bodies and floating ships. Two applications of these equations are developed in paragraphs 4 and 5. The one is of the problem of a deeply submerged and high speed body. This is not very interesting, but gives a mathematical foresight to the solution. The other is of the wave resistance of ships at very low speed. In the case of a hemisphere, the solution of the integral equation tells us the wave making character at very low speed is confined itself in the neighbourhood of the bow. Thus obtained resistance formula is very simple and seems to be applied to the ordinary ship at very low speed so that may be useful."

429:

Iordanskiĭ, S. V. The asymptotics of an axially symmetric diverging wave in a heavy fluid. Dokl. Akad. Nauk SSSR **125** (1959), 1211–1214 (Russian); translated as Soviet Physics. Dokl. **4**, 284–287.

The fluid is assumed to have a constant finite depth, and the motion should be such that a wave length is defined which is large compared with the depth. For such long waves the formula for the wave length of the well-known solitary wave is derived by making a development in powers of the depth coordinate. The same type of approximation is then used to discuss long waves with circular symmetry at long distances from the axis of symmetry, and with attention focussed on waves of the type of the solitary wave. In particular, a formula for the velocity of such an isolated "hump" is derived which is analogous to the formula for the solitary wave.

J. J. Stoker, Jr. (New York)

430:

Lauwerier, H. A. A note on the problem of the sloping beach. Nederl. Akad. Wetensch. Proc. Ser. A **62** = Indag. Math. **21** (1959), 229–240.

The problem of progressing waves over a uniformly sloping beach, first treated in the general case by Peters [Comm. Pure Appl. Math. **5** (1952), 87–108; MR **13**, 789] and Roseau [*Contribution à la théorie des ondes liquides de gravité en profondeur variable*, Publ. Sci. Tech. Ministère de l'Air, Paris, no. 275 (1952); MR **15**, 754] is solved by means of a representation which differs from those previously used. The new representation leads somewhat more directly to the solution, and may also be easier to work with in the discussion of the properties of the solution.

J. J. Stoker, Jr. (New York)

431:

Kostyukov, A. A. On the lifting force of source and dipole in bounded fluid stream. Prikl. Mat. Meh. **23** (1959), 389–391 (Russian); translated as J. Appl. Math. Mech. **23**, 547–550.

Asymptotic formulas are derived (from closed integral forms developed in a previous article) for the lift forces on a source, dipole or sphere moving in submerged motion at adequately extreme low and high speeds in a bounded fluid having a free surface. When the disturbances moving at the restricted speeds are less than 25 percent of the fluid depth from the free surface then the forces correspond to the deep fluid condition. The case when the disturbance is also near a vertical wall is similarly treated.

M. G. Scherberg (Dayton, Ohio)

432:

Mossakovskiĭ, V. I.; Rvačev, V. L. On the problem of horizontal hydrodynamic impact of a sphere. Prikl. Mat. Meh. **22** (1958), 847–849 (Russian); translated as J. Appl. Math. Mech. **22**, 1217–1220.

The problem considered is that of the horizontal hydrodynamic impact of (1) a sphere half-submerged in the free surface of a liquid, and (2) a sphere half-filled with a liquid. This note presents an analysis by which the solution of this problem may be obtained in a closed form. Suppose that the sphere, originally at rest, suddenly acquires a velocity U_0 along the x-axis which lies in the undisturbed free surface. The virtual mass coefficient λ_x of the sphere in this impulsive motion is defined by $\lambda_x = -P_t/(\frac{2}{3}\pi R^3 \rho U_0)$, where P_t is the x-component of the impulsive force acting on the wetted surface of the sphere, ρ is the density of the liquid and R is the radius of the sphere. The result shows that λ_x has the value $\lambda_x = (4/\pi) - 1$ for case (1) and $\lambda_x = (4/\pi) - 7/8$ for case (2). This result agrees numerically with the solution by È. L. Bloh which is expressed in the form of a series of spherical harmonics [Prikl. Mat. Meh. **17** (1953), 579–592; 705–726; MR **16**, 82].

T. C. Yao-tsu Wu (Pasadena, Calif.)

433:

Salamatov, Dž. Flow of liquid from an infinitely long axially symmetric container. Prikl. Mat. Meh. **23** (1959), 361–369 (Russian); translated as J. Appl. Math. Mech. **23**, 508–519.

This paper is concerned with the outflow of liquid from a container of conical form and with the form of the resulting jet. For related problems Zhukovskii has shown that the stream function can be expressed in terms of elliptic integrals. Following these ideas the author derives in great detail—under appropriate assumptions—similar expressions for the stream function of the present case. These expressions permit the numerical evaluation of the velocity distribution. Then an integral-equation for the form of the jet is solved by a process of successive approximations.

W. C. Rheinboldt (Syracuse, N.Y.)

434:

Budiansky, Bernard. Sloshing of liquids in circular canals and spherical tanks. J. Aero/Space Sci. **27** (1960), 161–173.

The normal modes of oscillation of a fluid in circular canals and spherical tanks are determined approximately from the numerical solutions of integral equations. The two-dimensional canal problem is treated for arbitrary depth of fluid while for the spherical tank only the nearly full and half-full cases are considered. Once the normal modes are determined the problem of sloshing of a fluid in a canal or tank is studied. Expressions are obtained for the amplitude of each mode in terms of the canal displacement and from these the sloshing forces can be determined. Some numerical examples are given.

R. C. MacCamy (Pittsburgh, Pa.)

435:

Babičev, A. I. Flows in a viscous liquid around oscillating surfaces. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 183–187. (Russian)

436:

Graffi, Dario. Sur un théorème d'unicité pour le mouvement d'un fluide visqueux dans un domaine illimité. C. R. Acad. Sci. Paris **249** (1959), 1741–1743.

In this paper, the author outlines a uniqueness proof for the solution of the equations of motion and continuity for an incompressible viscous flow exterior to a surface S, and satisfying four types of conditions: (1) the velocity v_j and its first derivatives in x_k ($k = 1, 2, 3$), t are continuous and bounded, with a finite number of discontinuities in their second derivatives; (2) the pressure p is continuous with first derivatives in x_k and converges to p_0 at infinity so that $p - p_0 = O(1/r)$; (3) v_j are given everywhere at $t = 0$ and on S for each t; (4) the components X_j of the external force are known for all permissible x_k, t. The method of proof consists in forming an integral which corresponds to the "difference motion". By use of the given conditions and inequalities of Cauchy and Schwartz, it is shown that only one motion is possible. Details are to appear in a future publication.

N. Coburn (Ann Arbor, Mich.)

437:

Rosenblat, S. Torsional oscillations of a plane in a viscous fluid. J. Fluid Mech. **6** (1959), 206–220.

The flow due to the torsional oscillations of a plane is shown to consist of two parts. There is first a primary boundary layer close to the plane, whose thickness is of the order of $(\nu/\lambda)^{1/2}$, where λ is the frequency of the oscillations, for which a solution is obtained by an expansion in the amplitude ε. The first-order solution involves only the transverse velocity and describes a simple unsteady shear layer. In the second-order solution there is both a steady term and a term of frequency 2λ. The steady term describes a radial-axial flow which, at large distances from the plane, has the form of an axisymmetric stagnation flow against an imaginary plane located a distance $(\nu/2\lambda)^{1/2}$ from the real plane. This is not the correct description of the steady flow, however; for, on closer examination, it is concluded that the expansion in powers of the amplitude is only valid inside the primary boundary layer and that a secondary boundary layer must exist in which inertia forces and centrifugal forces are of the same order of magnitude. The thickness of this secondary boundary layer is of the order of $\varepsilon^{-1}(\nu/\lambda)^{1/2}$ and may thus be large compared to the thickness of the primary boundary layer. It is further shown that the steady radial flow is confined to this secondary layer; far from the plane there is only a constant axial inflow as required by continuity.

W. H. Reid (Washington, D.C.)

438:

Elrod, H. G. A derivation of the basic equations for hydrodynamic lubrication with a fluid having constant properties. Quart. Appl. Math. **17** (1959/60), 349–359.

A refined form of Reynolds' lubrication equation is derived from Navier-Stokes equations through the technique of a small parameter ε which is the ratio of film thickness to a characteristic length of the bearing. The approximation error is of the order of ε^2. The derived equation is essentially Reynolds' lubrication equation with correction terms. In the case of slider bearings of flat surfaces these correction terms disappear. Reynolds' equation is again obtained. This indicates that Reynolds' equation of slider bearings has an error of $O(\varepsilon^2)$, rather than $O(\varepsilon)$. *L. N. Tao* (Chicago, Ill.)

439:

Kawaguti, Mitutosi. Reversal flow theorem concerning Oseen's approximation. J. Phys. Soc. Japan **14** (1959), 1431–1433.

440:

Kawaguti, Mitutosi. The flow of a viscous fluid past a triangular cylinder. J. Phys. Soc. Japan **14** (1959), 1425–1431.

The author calculates the flow of a viscous fluid past a triangular cylinder by Oseen's approximation.

Y. H. Kuo (Peking)

441:

Reid, W. H. The oscillations of a viscous liquid globe with a core. Proc. London Math. Soc. (3) **9** (1959), 388–396.

Author's summary: "The effect of viscosity on the gravitational oscillations of a liquid mantle covering a rigid core is considered. Detailed results for the aperiodic modes of decay are given for $l=1$, 2, 3, and 4, where l is the order of the spherical harmonic deformation considered, and for $\eta=\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ where η is the ratio of the core radius to the mantle radius."

R. C. DiPrima (Troy, N.Y.)

442:

Winter, Jacques. La théorie de la couche limite. Cahiers de Phys. **13** (1959), 353–362.

The paper is on physical and mathematical considerations which lead to the boundary layer theory and are well known in fluid dynamics. Two known examples.

I. Flügge-Lotz (Stanford, Calif.)

443:

Dem'yanov, Yu. A. Mixing of boundary layers at a liquid or gas interface. Prikl. Mat. Meh. **23** (1959), 370–375 (Russian); translated as J. Appl. Math. Mech. **23**, 520–527.

The problem considered is the flow of a viscous fluid past a semi-infinite flat plate which at time $t=0$ has viscosity $\mu_\pm$ and density $\rho_\pm$ for $x \gtrless 0$. A modified set of boundary layer equations are derived in which there appears the one additional parameter $\mu_-\rho_-/\mu_+\rho_+$. In the solution of the problem one must also determine the form of the dividing surface between the two fluids defined by $\eta=\Lambda(\xi)$, where $\eta=y(\rho_+V_\infty/\mu_+x)^{1/2}$ and $\xi=x/V_\infty t$. An approximate solution is obtained by using the momentum equation with polynomials of degree three. These results are then extended to the case of compressible fluids for which $\mu\rho=$ constant and their application to the problem of a shock wave running along the plate is briefly discussed.

W. H. Reid (Providence, R.I.)

444:

Skopec, M. B. Approximate method for integrating the equations of a laminar boundary layer in an incompressible gas in the presence of heat transfer. Ž. Tehn. Fiz. **29** (1959), 461–470 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 411–419.

The solution is based on the assumption that only small temperature differences occur and that the frictional heat can be neglected in the energy equation. Author transfers L. G. Loĭcyanskiĭ's method [Prikl. Mat. Meh. **13** (1949), 513–524; MR **11**, 477] of solving the velocity boundary layer equation to the temperature or energy equation. He develops a system of successive "moment" equations by multiplying the temperature equation with the kth power ($k=0$, 1, 2, $\cdots$) of the distance from the wall and then integrating each equation over the thickness of the thermal layer. He, finally, employs only the zero-th and first moment equations. By using families of velocity and temperature profiles from the flat plate flow and later from the wedge flow, author develops formulas for the Nusselt number of a flow over a surface with arbitrary pressure distribution. *I. Flügge-Lotz* (Stanford, Calif.)

445:

Soo, Shao L. Friction and heat transfer of compressible flow into an infinite lattice of flat plates. Z. Angew. Math. Phys. **10** (1959), 291–310. (German summary)

In this analysis the author considers the compressible flow of a viscous, heat conducting fluid past an infinite lattice of semi-infinite flat plates of zero thickness. The free stream has a uniform temperature and a steady velocity parallel to the plates, the lattice being unstaggered. Two cases of the wall temperature are considered: (1) constant wall temperature (idealizing the condition in a heat exchanger); (2) adiabatic wall (for the case of uncooled blade passage). The method can be extended, however, to the case of steady abitrary surface temperature distribution. The analysis is carried out with application to the classical laminar boundary layer theory with pressure gradient in the direction of flow, including the heat conduction and compressibility effects. The solution is obtained based on Howarth transformation and series expansion from the leading edge.

In the region where a core (the flow in between two adjacent boundary layers over two neighbor plates) exists, the presence of the neighbor plate gives rise to a pressure gradient in the flow direction, which is shown to increase with increasing Mach number. For moderate and large values of the Mach number, the pressure drop may become so large that the isentropic core exists only for a short distance from the leading edge. The acceleration of the flow in the core is found to have two significant effects. First, it results in higher friction drag than the single plate case, as should be expected on physical grounds. This increase is small for gap Reynolds number above 10^4 but is substantial for the Reynolds number below 10^4. Second, the acceleration of the core causes a reduction

of the temperature in the core. Consequently, the temperature along the insulated plates decreases from the leading edge at a greater rate than in the case of a single plate.

The high Mach number range considered in this study is seen to be an idealization since flow discontinuities are neglected altogether here. Moreover, no consideration is given for the flow further downstream where either the adjacent boundary layers may coalesce and converge to Poiseuille channel flow for the Reynolds number below the critical value or turbulent channel flow may be generated for higher Reynolds number, and any possible blockage effect associated with such change in the flow type. *T. Yao-tsu Wu* (Pasadena, Calif.)

446:

Philip, John R. Analysis of turbulent boundary layers with zero pressure gradient. Z. Angew. Math. Phys. **10** (1959), 478–501. (German summary)

A modified form of the mixing-length theory is used to provide a description of the mean flow in a turbulent boundary layer developing in zero pressure gradient. It is assumed that shear stress is related to velocity gradient by $\tau/\rho = ky^m(\partial u/\partial y)^2$, where m is a constant and k is a function of relative position in the layer. Self-preserving flow is assumed with a velocity scale equal to the stream velocity U and a length scale of $x^{1/(3-2m)}$, and the function k and the value of m are determined by inserting experimental measurements of velocity in the resulting ordinary equation. Using these values reasonable agreement is found between the predicted profiles of velocity and shear stress and the experimental values, but the analysis, though probably convenient for computation, takes little account of the physical meaning of self-preserving flow and ignores the unified treatment of flow over rough and smooth surfaces that is possible once the relevance of the friction velocity is understood.

A. A. Townsend (Cambridge, England)

447:

Wasserman, Robert H. Helical fluid flows. Quart. Appl. Math. **17** (1959/60), 443–445.

The author determines all flows (incompressible and compressible) of a perfect fluid whose streamlines are helices on a given right circular stream cylinder (the pitch may vary from cylinder to cylinder). Since such helices satisfy the condition that the divergence of the unit tangent vector vanishes, the author considers an intrinsic form of the equations of motion, continuity, and energy. That is, these equations are expressed in terms of directional derivatives of density, ρ, pressure, p, and magnitude of velocity, q, along the tangent, principal normal, and binormal of the streamlines, and the curvature of these lines. Since helices are geodesics, the principal curvature of the streamlines can be obtained by Euler's theorem and the variation of pressure with respect to radius of the cylinder, r, can be explicitly determined. The flows can be classified into two types: (1) ρ, p, q are constant along any given streamline and the ordinary polytropic gas law is valid, and (2) a generalized Karman-Tsien pressure-volume relation is valid. Most cases are discussed completely. Typical of the results stated is: p is a non-decreasing function of r, and hence if the entropy and stagnation enthalpy are constant, then the Mach number must decrease with r. *N. Coburn* (Ann Arbor, Mich.)

448:

Naylor, D. Unsteady rectilinear gas flow. J. Math. Mech. **9** (1960), 1–18.

The equations of one-dimensional gas flow imply the existence of $\psi(x, t)$ such that $\psi_x = a^n$ and $\psi_t = -ua^n$ where a is speed of sound and $n = 2/(\gamma - 1)$. Transform the equations to Lagrangian form to obtain

$$\text{(i)} \qquad x_\psi^{\gamma+1}x_{tt} - x_{\psi\psi} = x_\psi S'(\psi)/\gamma R,$$

where $S(\psi)$ is specific entropy. Let $S = R \log g'(\psi)$, observe that (i) can then be written in divergence form, and exploit this fact to define a new function $v(\psi, t)$. Now let $\lambda = u_t$ and $w(v, \lambda) = u - t\lambda$. Then (i) implies

$$\text{(ii)} \qquad w_{vv} - \lambda^{(\gamma+1)/\gamma} f(w_v) w_{\lambda\lambda} = 0,$$

where since $w_v = u_v = 1/\gamma g(\psi)$, f is an elementary function of $g(\psi)$ and $g'(\psi)$. By imposing the condition that (ii) have fixed characteristics the author obtains $g^{n+2} = \psi_0/\psi$ where ψ_0 is a constant, and now t satisfies an Euler-Poisson-Darboux equation, as G. S. S. Ludford and M. H. Martin have shown [Comm. Pure Appl. Math. **7** (1954), 45–63; MR **16**, 192]. Also considered are degenerate solutions of (i) for which x_ψ and x_t are functionally dependent; now

$$\text{(iii)} \qquad g(\psi) = (\psi_0/\psi)^m$$

for any m, and x, t, and u can be found explicitly as functions of the parameter a. Furthermore, for g of form (iii) equation (i) has non-degenerate solutions of the form $x = tk(\psi) + H(t)$ for which the general forms of k and H have been determined. The case $\gamma = -1$ for (i) is also briefly considered. *J. H. Giese* (Aberdeen, Md.)

449:

Ginsburg, Th. Ausbreitung und reflexion von starken Explosionswellen. Symposium on the numerical treatment of partial differential equations with real characteristics: Proceedings of the Rome Symposium (28–29–30 January 1959) organized by the Provisional International Computation Centre, pp. 145–156. Libreria Eredi Virgilio Veschi, Rome, 1959. xii + 158 pp.

450:

Chang, C. C.; Lundgren, T. S. Airfoil in a sonic shear flow jet: a mixed boundary value problem for the generalized Tricomi equation. Quart. Appl. Math. **17** (1959/60), 375–392.

The author considers a parallel but non-uniform stream of compressible fluid perturbed by a thin two-dimensional airfoil which is located along a sonic line. The main stream Mach number distribution $M(y)$ is assumed symmetrical about the sonic line at $y = 0$. It is generally accepted that the differential equation

$$\text{(*)} \qquad (1 - M^2)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - \frac{2}{M}\frac{dy}{dM}\frac{\partial\phi}{\partial y} = 0$$

is a valid linearization of the equations of fluid motion provided that dM/dy is large enough at points where $M = 1$. In general, solutions of (*) are difficult to obtain with any arbitrarily given $M(y)$. However for a particular

choice of $M(y)$ (*) can be transformed to the Tricomi equation

$$Y^n \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0, \quad Y \gtrless 0, \quad n \gtrless 0,$$

$$Y = b\int_0^y M^2(y)dy, \quad b = \text{const.}$$

The problem is to solve the generalized Tricomi equation with the mixed boundary conditions

$$\phi(x, 0) = 0, \quad x < 0; \quad \phi(x, 0) = -\frac{C}{2}, \quad x > 1;$$

$$\frac{\partial\phi(x, 0)}{\partial Y} = \Theta(x), \quad 0 < x < 1; \quad \phi(x, \infty) = 0.$$

Here C is a constant and $\Theta(x)$ is bounded and integrable. If $\phi(x, 0)$ is given along the whole x-axis the problem is a Dirichlet problem which can be solved by use of Fourier integrals or two-sided Laplace transforms. The author solves the related auxiliary problem, namely the generalized Tricomi equation with $\partial\phi(x, 0)/\partial Y$ specified on the positive x-axis, and $\phi(x, 0) = 0$ on the negative x-axis. This is done with the aid of two-sided Laplace transforms and the method of Wiener-Hopf. In the main problem, $\partial\phi(x, 0)/\partial Y$ is given on $(0, 1)$ and $\phi(x, 0)$ is specified on $(1, \infty)$, consequently, the auxiliary problem gives an integral equation for the unknown quantities $\phi(x, 0)$ on $(0, 1)$ and $\partial\phi(x, 0)/\partial Y$ on $(1, \infty)$. This integral equation is solved by using Mellin transforms with the Wiener-Hopf technique. The result reduces the original mixed boundary value problem to a much simpler Dirichlet problem. The airfoil in a sonic shear flow jet is considered in part II. Certain properties of the upstream Mach number distribution $M(y)$, which are defined by the equation

$$\frac{1-M^2}{b^2M^4} = Y^n = b\int_0^y M^2dy, \quad n \gtrless 0,$$

can be enumerated without completely solving this equation. Finally the pressure coefficient, lift coefficient and center of pressure are calculated in some special cases.

M. Pinl (Cologne)

451:

Grodzovskiĭ, G. L. Supersonic axisymmetric conical flows with conical shocks adjacent to uniform parallel flows. Prikl. Mat. Meh. **23** (1959), 379–383 (Russian); translated as J. Appl. Math. Mech. **23**, 532–538.

Two new types of solution of the supersonic conical flow field equations are considered for regions separated from a uniform flow by a shock. The new solutions, corresponding to flow in suitably shaped channels, describe a converging flow behind the shock and a diverging flow ahead of the shock. Tabulated solutions are given in five cases.

J. J. Mahony (Sydney)

452:

Khaskind, M. D.; Khomenko, V. S. Flow past a profile in a bounded supersonic stream. J. Appl. Math. Mech. **22** (1958), 1162–1168 (815–818 Prikl. Mat. Meh.).

Two-dimensional, steady, irrotational and homentropic flow past a sharp-nosed aerofoil between parallel flat walls is studied by the help of an approximate equation of state for the gas.

As in most cases where this approach is used, the authors fail to realise that it is a form of self-deception. Despite apparently close agreement between the gas laws, exact solutions for fictitious gases have been shown repeatedly [Mahony and Meyer, Philos. Trans. Roy. Soc. London Ser. A **248** (1956), 467–498; MR **17**, 913; Mackie, Proc. Cambridge Philos. Soc. **54** (1958), 538–553; MR **20**#3700] to be equivalent only to low-order approximations in perturbation schemes with respect to thickness and incidence parameters for the perfect gas.

The authors' gas law reduces the equations of motion to the wave equation for the Legendre potential, and so their solution is only a linear perturbation on a uniform stream for the perfect gas. It is the wave equation in characteristic independent variables, however, and so they obtain a uniform first-order approximation, rather than mere linearised theory. An example indicates that this uniformisation has a very marked effect on the lift and moment coefficients.

A realistic treatment of the problem requires at least a consideration of the shocks. A method for such treatment, correct even to second order, is due to Mahony [Philos. Trans. Roy. Soc. London Ser. A **248** (1956), 499–515; MR **17**, 914].

R. E. Meyer (Providence, R.I.)

453:

Maĭkapar, G. I. On the wave drag of non axisymmetric bodies at supersonic speeds. Prikl. Mat. Meh. **23** (1959), 376–378 (Russian); translated as J. Appl. Math. Mech. **23**, 528–531.

An inverse method is used to obtain the solution for supersonic flow past a family of non-slender bodies. The family consists of pyramids, whose cross-sectional shape is a re-entrant polygon, such that the flow consists of uniform flows behind plane shock waves joining neighbouring ridges of the pyramid. Expressions are given for the drag and numerical comparisons are made with the drag of right circular cones of the same base area.

J. J. Mahony (Sydney)

454:

Vorob'ev, N. F. Non-stationary motion of a wing of finite span in supersonic flow with jump discontinuities in velocity. Izv. Akad. Nauk SSSR Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 167–170. (Russian)

455:

Collar, A. R. Iterative solutions of the equations for plane oblique shock waves. J. Roy. Aero. Soc. **63** (1959), 669–672.

[This paper was previously listed as MR **21** #4718.]

The angle formed by a shock wave in the neighborhood of a wedge may be expressed in terms of a cubic. The author discusses a convenient method for solving this cubic.

H. Polachek (Washington, D.C.)

456:

Grigoryan, S. S. Motion of a thin solid body under the action of a strong shock wave. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 165–166. (Russian)

457:

Grodzovskii, G. L.; Diukalov, A. N.; Tokarev, V. V.; Tolstyh, A. I. Selfsimilar motions of gas with shock waves, spreading with constant speed into gas at rest. J. Appl. Math. Mech. **23** (1959), 281–286. (198–200 Prikl. Mat. Meh.).

458:

Kočina, N. N. On particularities near the detonation center and on the appearance of two shock waves. Dokl. Akad. Nauk SSSR **126** (1959), 1216–1219 (Russian); translated as Soviet Physics. Dokl. **4**, 544–548.

Similarity solutions for detonation waves are available when the medium has an equation of state of a suitable 'similarity form'. The author considers the case when the equation of state differs but little from this 'similarity form' and the difference is of a specified functional form. By neglecting terms of second order in the small difference he obtains a set of ordinary differential equations relevant for detonation waves in the new medium. A brief qualitative description of the properties of solutions of these equations is given. *J. J. Mahony* (Sydney)

459:

Rues, Dietrich. Der Einfluss ebener Körper auf stromaufwärts liegende senkrechte Stösse. Z. Flugwiss. **8** (1960), 67–76. (English and French summaries)

Author's summary: "Under the assumption of small perturbations the influence of two-dimensional bodies on normal stationary shocks is investigated. As an example, the numerical values for the flat plate at incidence and the parabolic biconvex airfoil are calculated. A simplified method is developed for symmetrical profiles and compared with the preceding results."

460:

Sen, S. N. Hypersonic velocity in viscous liquids as revealed from Brillouin spectra. Proc. Phys. Soc. **75** (1960), 612–616.

Author's summary: "Taking into consideration Lamb's theory for the propagation of high frequency acoustical waves in viscous liquids, and the idea of Maxwell regarding complex viscosity, a theoretical expression has been deduced for the velocity of high frequency acoustical waves through viscous liquids. It is observed that if Frenkel's expression for the rigidity modulus of a liquid be introduced into the formula for wave propagation then the theoretical formula explains the dispersion of sound velocity in viscous liquids as was observed by Venketeswaran from Brillouin spectra and the theoretical value is in good agreement with the experimental result. Further, the introduction of complex viscosity partly explains the propagation of these high frequency waves through viscous liquids."

461:

Liénard, Pierre. Étude et réalisation d'une base élastique a raideur variable pour isolement des vibrations. Ann. Télécommun. **15** (1960), 61–70.

Résumé de l'auteur: "Rappel des principes d'isolation des vibrations par supports élastiques. Utilisation des propriétés d'élasticité non linéaire du caoutchouc et recherche d'une forme d'appuis courbes pour obtenir une raideur croissante avec la charge. Étude expérimentale d'une qualité de caoutchouc et conclusions pour sa mise en œuvre."

462:

Polyakova, A. L. Thermodynamic theory of the absorption of finite-amplitude sound in relaxing media. Soviet Physics. Acoust. **5** (5) (1959), 85–90 (85–90 Akust. Ž.).

The paper discusses propagation and absorption of sound in a medium in which internal processes are possible, e.g., such as those connected with the internal energy of the molecules and the transfer of energy back and forth between molecular oscillations. The method of treatment is that of irreversible thermodynamics. Amplitudes are allowed to be finite. In the passage of sound, the particles of the medium can be transferred to excited states where the oscillatory or rotatory degrees of freedom appear. The change in the number of excited particles is shown to be due to displacement of these particles with hydrodynamic velocity and to the reaction of the transition to an excited state and back again. An expression for the energy dissipation is derived and the differences in the absorption of sound of finite and infinitesimal amplitudes are discussed. *B. Gross* (Rio de Janeiro)

463:

Masterov, E. P. Sound channel propagation in laminar-inhomogeneous media. Akust. Ž. **5** (1959), 332–336 (Russian); translated as Soviet Physics. Acoust. **5** (1960), 339–343.

A biexponential law, $n^2(z)=p^2+(1-p^2+q)e^{-az}-qe^{-2az}$, is proposed for the variation of the acoustic index of refraction in a stratified-inhomogeneous medium, $z \geqq 0$. The parameters p, q, a are given functions of c_0, c_m, c_∞, the velocities of sound at $z=0$, $z=z_m$, and for $z \to \infty$, respectively, where z_m is a value of z at which c is an extremum. Equations of the rays, solution of the wave equation, and study of the distant field by splitting into normal and lateral waves are taken up successively. *R. N. Goss* (San Diego, Calif.)

464:

Hattori, Tsuyoshi. A treatise on the screw extrusion of thermoplastics. I. Theory of the screw extrusion of non-Newtonian liquid. Bull. JSME **2** (1959), 475–480.

A one-dimensional analysis of flow of non-Newtonian fluids through a screw extruder. *J. L. Ericksen* (Baltimore, Md.)

465:

Steginsky, Bernard. Magnetohydrodynamics. Battelle Technical Review **9** (1960), no. 2, 3–9.

466:

Kovrižnyh, L. M. Stability of a hollow gas conductor in a magnetic field. Ž. Èksper. Teoret. Fiz. **37** (1959), 92–94 (Russian); translated as Soviet Physics. JETP **10** (1960), 65–66.

Author's summary: "Stability conditions are obtained

for a hollow gaseous conductor located in a longitudinal magnetic field."

467:

Lüst, R. Some theoretical aspects of magneto-hydrodynamics and thermo-nuclear fusion. Nuovo Cimento (10) **13** (1959), supplemento, 330–335.

This paper contains a brief outline of three topics in magneto-hydrodynamics: (1) magnetostatic equilibrium; (2) the stability of equilibria treated by the energy-principle; (3) charge trajectories in a given magnetic field.

N. L. Balazs (Princeton, N.J.)

468:

Ryazanov, E. V. Some exact solutions of the equations of magneto-gas dynamics with internal gravitational forces present and with a zero temperature gradient. Dokl. Akad. Nauk SSSR **126** (1959), 1224–1226 (Russian); translated as Soviet Physics. Dokl. **4**, 554–557.

469:

McCune, J. E.; Sears, W. R. On magnetohydrodynamic channel flow. J. Aero/Space Sci. **27** (1960), 139–140.

It is the authors' intention in the present note "to clear up certain confusion" which is believed to exist concerning the approximations made in a preceding paper, by Resler and Sears [J. Aero Sci. **25** (1958), 235–245, 258; Z. Angew. Math. Phys. **9b** (1958), 509–518; MR **19**, 1226; **20** #616]. (The sources of confusion are also referenced.)

H. Greenspan (Cambridge, Mass.)

470:

Lyubimov, G. A. Stationary flow of an ideally conducting gas around a corner. Dokl. Akad. Nauk SSSR **126** (1959), 733–735 (Russian); translated as Soviet Physics. Dokl. **4**, 529–531.

The flows considered are those in which the physical variables are functions of φ alone in the cylindrical coordinate system r, φ, z. Conductivity is infinite. First there are presented three solutions in which pressure and density are constant. Next there are homentropic flows about corners, with magnetic and velocity vectors everywhere parallel. Both infinitely conducting and non-conducting corners are treated. In the latter there are two possible solutions, in which the velocity component v_φ is equal to the fast and slow magnetohydrodynamic sound speeds, respectively. In the first of these the waves are inclined downstream; in the second, upstream. This is only a concise summary of the work, with very little detail given.

W. R. Sears (Ithaca, N.Y.)

471:

Kiselev, M. I. On the calculation of shock waves in magnetohydrodynamics. Dokl. Akad. Nauk SSSR **126** (1959), 524–527 (Russian); translated as Soviet Physics. Dokl. **4**, 517–520.

This is a concise summary of some results on shock waves in ideal gaseous conductors. Details are not given here. First, formulas are given for conditions behind a normal shock with magnetic field parallel to the front. Second, these are used to write shock-polar equations; it is shown that the shock polar diminishes in size with increasing field and the maximum turning angle decreases. For other orientations of the field relative to a normal-shock front a coordinate translation can always reduce the problem to one of plane flow with velocity and magnetic vectors lying in the same plane on both sides of the front. A result for this case is stated. Finally, a geometrical interpretation in a certain velocity space is given for the shock transition in the plane one-dimensional case.

W. R. Sears (Ithaca, N.Y.)

472:

Bush, William B. Compressible flat-plate boundary-layer flow with an applied magnetic field. J. Aero/Space Sci. **27** (1960), 49–58.

This paper presents numerical results (machine calculations) for the skin friction and heat transfer characteristics obtained from the laminar boundary layer equations governing the high speed compressible, ionized air flow (in equilibrium dissociation) past a flat plate in the presence of an applied magnetic field whose component normal to the plate is proportional to $1/\sqrt{x}$. The major results are as follows. (1) The skin friction and heat transfer decrease with increasing magnetic field strength for constant values of the free stream velocity. The reduction in heat transfer is opposite in behavior from that determined in Couette flow. (2) The foregoing quantities also exhibit an hysteresis effect found by Bleviss [J. Aero. Sci. **25** (1958), 601–615; MR **20** #2966].

H. Greenspan (Cambridge, Mass.)

OPTICS, ELECTROMAGNETIC THEORY, CIRCUITS

See also A49, 258, 467, 538, 614.

473:

Biot, A. La lentille épaisse convergente. I. Représentation graphique et chromatisme des longueurs focales. Ann. Soc. Sci. Bruxelles. Sér. I **73** (1959), 345–353.

Graphical methods are described for making a systematic study of the single thick positive lens with respect to longitudinal chromatic aberration.

E. W. Marchand (Rochester, N.Y.)

474:

Biot, A. La lentille épaisse convergente. II. Achromatisme du point image sur l'axe. Ann. Soc. Sci. Bruxelles. Sér. I **73** (1959), 354–362.

The methods of Part I [see preceding review] are used to investigate the case in which an axial object point is achromatically imaged.

E. W. Marchand (Rochester, N.Y.)

475:

Biot, A. L'épaisseur équivalente d'une lentille absorbante. Ann. Soc. Sci. Bruxelles. Sér. I **73** (1959), 363–372.

Formulas are derived for the total absorption of a thick lens. It is assumed that the inclination of the rays with the axis may be neglected.

E. W. Marchand (Rochester, N.Y.)

476:

Rosendahl, Gottfried R. General relation between object and image space in plane-mirror optics. J. Opt. Soc. Amer. **50** (1960), 287–289.

Formulas are given for optically transferring an object into a specified position by means of two plane mirrors. The results can be used, for example, to design an efficient illumination system for a microcard reader.

E. W. Marchand (Rochester, N.Y.)

477:

Marathay, A. S. Geometrical optical calculation of frequency response for systems with coma. Proc. Phys. Soc. **74** (1959), 721–730.

Author's summary: "The usefulness of the geometrical optical treatment of frequency response is considered here for the case of coma, an aberration having odd symmetry. Calculations of frequency response and phase shift using this treatment have been made, for different orientations of incoherent line objects, in the presence of Seidel coma in different focal planes. Similar calculations have been made for higher order coma in the Gaussian focal plane. Results are also given according to a mixed treatment of frequency response. The results are compared with diffraction calculations made by Goodbody. It appears from these results that the mixed treatment leads to a smaller value of the response than that given by diffraction theory, and that the geometrical optical treatment will give errors less than about 5% when the wave-front aberration exceeds about 1.5 times the Strehl type of tolerance."

478:

Longuet-Higgins, M. S. The focusing of radiation by a random surface when the source is at a finite distance. Proc. Cambridge Philos. Soc. **56** (1960), 27–40.

From the author's summary: "Imagine a nearly horizontal, statistically uniform, random surface $\zeta(x, y)$, Gaussian in the sense that the second derivatives $\partial^2\zeta/\partial x^2$, $\partial^2\zeta/\partial x\partial y$, $\partial^2\zeta/\partial y^2$ have a normal joint distribution. The problem considered is the statistical distribution of the quantity

$$F = 1 + \nu J + \nu^2\Omega,$$

where J and Ω denote the mean curvature and total curvature of the surface, respectively, and ν is a constant parameter.

"The problem is solved completely when the surface is statistically isotropic; typical curves for the distribution of F are shown. From this distribution we deduce the distribution of the sizes of images reflected in the surface."

R. A. Silverman (New York)

479:

Meiron, J.; Volinez, G. Parabolic approximation method for automatic lens design. J. Opt. Soc. Amer. **50** (1960), 207–211.

In order to use automatic methods to minimize a merit function for an optical system it is suggested that one begin by varying only one of the lens parameters at a time and use a parabolic approximation to locate the corresponding minimum point. Then the minimum of the minima so obtained gives an improved system as compared to the original one.

Although slower to compute, the proposed method appears to have certain advantages over the least squares and steepest descent methods. A comparison of methods is illustrated in the case of a Cooke triplet.

E. W. Marchand (Rochester, N.Y.)

480:

Boerboom, A. J. H. A modified ion slit lens for virtual variation of slit widths. Z. Naturf. **15a** (1960), 350–355.

Author's summary: "It is shown that a slit lens system, already known in literature and used in mass spectrometer collector systems to vary the resolving power, can be greatly improved by introducing a potential on one of the electrodes, which previously was at zero potential. The focusing at the collector is unaltered within optical aberrations of the third order and the range of adjustment is increased as compared with the original system. Experimentally it was found that the virtual collector slit width could be adjusted from 1 mm down to 0.15 mm, maintaining a fair peak shape."

481:

Vorob'ev, Yu. V. Image defects of cathode electron lenses with violation of axial symmetry of the field. Ž. Tehn. Fiz. **29** (1959), 589–596 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 525–531.

This paper deals with focusing and image defects in a system where the usual paraxial treatment is inapplicable due to the width of the electron beams. A moving system of rectangular coordinates is used in deriving the equations for the trajectories. These equations are integrated by a series method. Explicit expressions are given for first order aberrations, and a study is made of the corresponding image properties. Since the expressions for second order aberrations are cumbersome, this problem is studied only for cases where no less than two planes of symmetry are present.

J. E. Rosenthal (Passaic, N.J.)

482:

Faĭnberg, Ya. B. Variable-phase focusing in linear accelerators. Ž. Tehn. Fiz. **29** (1959), 568–579 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 506–516.

Author's abstract: "A method of variable-phase focusing for linear accelerators is proposed and investigated. The phase and radial motions are considered. The question of the relation between the radial and the phase motion is investigated, and other possible applications of the method of variable-phase focusing are indicated."

J. E. Rosenthal (Passaic, N.J.)

483:

Dreicer, H. Electron velocity distributions in a partially ionized gas. Phys. Rev. (2) **117** (1960), 343–354.

The Boltzmann method is applied to the problem of calculating the electron velocity distribution in a partially ionized gas. Inelastic collisions with neutral molecules as well as random two-body Coulomb interactions are included. The latter are treated by use of the Fokker-Planck equation. Application is made to a hydrogen plasma subjected to an externally applied electric field. Static solutions are obtained, by numerical means, as a function of the ionization degree, and the classical gas

discharge parameter, E/p. It is shown that the evolution of the electron velocity distribution function from that characteristic of a poorly ionized gas to the Maxwellian distribution occurs over a very large range in ionization degree. Several applications are made to energy relaxation phenomena, and the electrical conductivity is evaluated. Excitation and ionization rates are also given and it is shown when these may be obtained from an assumed Maxwellian distribution. *W. P. Allis* (Cambridge, Mass.)

484:

Fainberg, Ya. B.; Gorbatenko, M. F. Electromagnetic waves in a plasma situated in a magnetic field. Ž. Tehn. Fiz. **29** (1959), 549–562 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 487–500.

In this work Maxwell's equations and the two-fluid transport equations are solved to investigate the propagation of electromagnetic waves in a plasma rod. The plasma is situated in an external, unvarying, homogeneous magnetic field directed along the rod, and is isolated from metallic walls. Expressions for the electric and magnetic vectors are obtained in terms of Bessel, Neumann and Hankel functions are obtained, and hence the dispersion equation is written down. The solutions are discussed in terms of regions in the plane of the parameters ω_H^2/ω^2 and ω_0^2/ω^2 and numerical solutions are given for some cases. Only high frequency waves such that ion motions may be neglected are studied, and collisions are neglected. *W. P. Allis* (Cambridge, Mass.)

485:

Ferraro, V. General theory of plasma. Nuovo Cimento (10) **13** (1959), supplemento, 9–58.

The purpose of these lectures is to give an elementary discussion of the dynamics of ionized gases following the macroscopic method of the kinetic theory of gases. The notation of Chapman and Cowling is adopted. The Boltzmann equation is written down but is little used as the distribution function is assumed to be slightly perturbed Maxwellian and the collision term is represented by a relaxation time $\tau(C)$. The density—and momentum—transport equations for a two-component (completely ionized) gas are then derived and used throughout the lectures. The pressure tensors are assumed to reduce to two hydrostatic pressures. The electrical and thermal conductivities are then derived for a plasma at rest, both without and with a magnetic field. Unfortunately a completely ionized plasma in crossed electric and magnetic fields is not at rest for very long, so in this section the relaxation time must be assumed to be due to collisions with neutrals. Two vorticity theorems are proved. Waves in a plasma are then considered extremely briefly.
W. P. Allis (Cambridge, Mass.)

486:

Niblett, G. B. F.; Green, T. S. Radial hydromagnetic oscillations. Proc. Phys. Soc. **74** (1959), 737–743.

Authors' summary: "This paper discusses the radial hydromagnetic oscillations of a plasma confined by an axial magnetic field. Oscillations of this type have recently been observed experimentally in this laboratory and typical high speed streak photographs are presented and analysed. On the assumption that the plasma is confined in a thin cylindrical annulus, the non-linear equation of motion can be integrated analytically. The calculated period of the oscillations is independent of amplitude and is found to be in good agreement with experimental results. Damping of the oscillations by diffusion of the magnetic field is discussed and reference made to the possible significance of the oscillations as a means of transferring energy irreversibly to the ions."

487:

Körper, Klaus. Schwingung eines Plasmazylinders in einem axialen Magnetfeld. II. Z. Naturf. **15a** (1960), 220–226. (English summary)

[For part I, see same Z. **12a** (1957), 815–821; MR **19**, 1227.]

Author's summary: "Radial oscillations are excited in a homogeneous infinite plasma cylinder in a homogeneous axial magnetic field by a surface current which is homogeneous in the axial and azimuthal directions. The modes of oscillations corresponding to the axial and azimuthal components of current are not coupled, and so they may be analysed separately. The magnetic field in the plasma and vacuum is obtained, and the indices of refraction for both types of oscillations are discussed thoroughly. When the currents are parallel to the external magnetic field, the oscillations are characterized by the refractive index of Eccles. On the other hand, when the current is perpendicular to the magnetic field two resonance frequencies exist, which depend on the density of the plasma and the magnetic field strength.—In the latter case the radial characteristic oscillations of the plasma cylinder in an external magnetic field are considered."

488:

Körper, Klaus. Die Impedanz einer Spule mit einem Plasma als Dielektrikum. Z. Naturf. **15a** (1960), 226–235. (English summary)

Author's summary: "From the basic equations of magneto-hydrodynamics the energy conservation theorem for plasmas is derived for events periodic in time. In addition to the terms known from vacuum electrodynamics, there are the kinetic energies of the electrons and ions, and a term due to the oscillations of the electrons and ions in the magnetic field.—With the help of the solution for the radially oscillating cylinder, the impedance of a coil containing the plasma is derived from general energy considerations. The impedance is discussed and its mean value in the range between two eigenfrequencies of the plasma cylinder is given."

489:

Körper, Klaus. Hochfrequenzheizung eines Plasmazylinders in einem axialen Magnetfeld. Z. Naturf. **15a** (1960), 235–243. (English summary)

Author's summary: "The conditions under which the effect of ion-electron-scattering on radial oscillations near the ion-resonance of an infinite plasma cylinder in an axial magnetic field may lead to a heating of the plasma, possibly sufficient to induce thermonuclear reactions, are examined.

"The magnetic field strengths needed to stabilize the plasma, and the density of the plasma necessary to

guarantee a sufficient rate of reaction require that we consider the optical behaviour in the decimeter-wavelength-range.

"In an equivalent model for the supplying circuit the reaction of the plasma is taken into account in a quantitative manner.

"Because of the finite extent of the plasma a dense spectrum of eigenresonances of the plasma system exists. Therefore the matching of the external circuit to the plasma is possible only in the mean.

"Numerical results are given for the energy absorbed, the radiative energy penetrating the plasma cylinder, and the corresponding reactive output, as functions of the plasma density, the temperature and the frequency (near the "ion-resonance" = geometric mean value between the gyrofrequencies of the ions and electrons), when the oscillator and the plasma are optimally matched."

490:

Adlam, J. H.; Allen, J. E. Collision-free hydromagnetic disturbances of large amplitude in a plasma. Proc. Phys. Soc. **75** (1960), 640–648.

Authors' summary: "Calculations are given of certain collision-free hydromagnetic disturbances produced by the rapid compression of a plasma containing a magnetic field."

491:

Poincelot, Paul. Critère d'unicité pour les solutions des équations de Maxwell. Ann. Télécommun. **15** (1960), 77–83.

Résumé de l'auteur: "En électromagnétisme théorique, de nombreux problèmes de propagation, de diffraction et de rayonnement se ramènent à l'intégration des équations de Maxwell auxquelles on impose des conditions aux limites. On recherche généralement une solution particulière, dont la validité n'est pas certaine a priori; l'expérience a montré qu'il pouvait en résulter des erreurs. En s'inspirant d'un grand nombre de travaux antérieurs, publiés par des mathématiciens et des physiciens, l'auteur établit un critérium d'unicité valable pour les surfaces lisses ou présentant des arêtes."

492:

Borgnis, F. Zur elektromagnetischen Energiedichte in Medien mit Dispersion. Z. Physik **159** (1960), 1–6. (English summary)

Author's summary: "General expressions for electromagnetic energy densities and losses in dispersive media are derived. They depend in general on the way the fields have been established. Only when fields of harmonic time dependence are built up with extreme slowness the energy becomes independent of the transient character of the field and it can be represented by a simple expression. An example sheds light on the nature of energies involved."

493:

Rutkevič, B. N.; Faĭnberg, Ya. B. The propagation of oscillations in electron beams with uncompensated space charge. Ž. Tehn. Fiz. **29** (1959), 280–286 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 246–253.

A study is made of the propagation of electromagnetic waves in an electron cloud rotating in a uniform magnetic field. The mathematical calculations are presented in detail. Conditions are found for the propagation of waves with frequencies in a certain range and for electron beam stability. *J. E. Rosenthal* (Passaic, N.J.)

494:

Fischer, J. Über die Anzahl der von einander unabhängigen Grössen in der Elektrizitätslehre. Arch. Elektrotech. **45** (1960), 77–98.

495:

Durand, Philippe. Développement en série du potentiel cylindrique au voisinage de la limite d'une distribution superficielle de charges et recherche d'une équation aux différences finies. C. R. Acad. Sci. Paris **249** (1959), 1864–1866.

496:

Smythe, W. R. Charged sphere in cylinder. J. Appl. Phys. **31** (1960), 553–556.

The method first used by the author to solve the problem of a freely charged right circular cylinder of finite length [J. Appl. Phys. **27** (1956), 917–920; MR **18**, 169] is applied to the problem of a sphere maintained at a constant potential and enclosed by an infinite earthed coaxial circular cylinder. Expressions for the charge density on the sphere and the potential at any point are given as infinite series, certain of the coefficients in these series together with the capacitance of the configuration being evaluated numerically for various values of the sphere to cylinder radius ratio. An approximate formula for the capacitance of a sphere in a cylindrical box is given. *W. D. Collins* (Newcastle-upon-Tyne)

497:

Boerboom, A. J. H. The potential distribution in a toroidal condenser. Z. Naturf. **15a** (1960), 347–350. (English summary)

Author's summary: "The electrostatic potential is calculated in a toroidal condenser consisting of two rotational symmetric electrodes having the median plane as a plane of symmetry. The result is expressed as a series expansion in the coordinates around the main circle and in the mutual distance of the electrodes in the median plane."

498:

Cyrlin, L. È. The unipolar corona in the case of an eccentric cylinder. Ž. Tehn. Fiz. **29** (1959), 563–567 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 501–505.

This problem in potential theory involves the finding of the field between two non-coaxial parallel conducting cylinders, one inside the other. A perturbation method is used when the ratio of displacement of the inner conductor from the centre of the outer, to the radius of the outer conductor is small.

V. M. Papadopoulos (Providence, R.I.)

499:

Weinstein, M. A. On the validity of Kirchhoff's law for a freely radiating body. Amer. J. Phys. **28** (1960), 123–125.

500:

Tihonov, A. N. The propagation of a continuous electromagnetic wave in a laminarly anisotropic medium. Dokl. Akad. Nauk SSSR **126** (1959), 967–970 (Russian); translated as Soviet Physics. Dokl. **4**, 566–570.

The author is concerned about the electromagnetic field on the plane surface of an anisotropically conducting medium in a half-space, this field being produced by a given current distribution in the surface. The conductivities normal and parallel to the plane are piecewise smooth functions of the depth.

He demonstrates a technique involving integral transforms giving results suitable for electronic computation in the anisotropic case.

V. M. Papadopoulos (Providence, R.I.)

501:

Wait, James R. On the electromagnetic response of a conducting sphere to a dipole field. Geophysics **25** (1960), 649–658.

Author's summary: "The electromagnetic coupling between an electric and a magnetic dipole in the presence of a spherical ore body is discussed in the present paper. It is shown that both electric and magnetic modes must be considered even though all dimensions are small compared to the wave length. The results have application to a geophysical prospecting scheme where the exciting fields are set up by a straight insulating wire grounded at its end points and the secondary fields are detected by an ungrounded closed wire loop."

502:

Kaner, È. A.; Bass, F. G. Toward the statistical theory of radiowave propagation over an ideally conducting plane. Dokl. Akad. Nauk SSSR **127** (1959), 792–795 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 847–850.

The authors treat (in the customary compressed Doklady style) the problem of the propagation of electromagnetic waves in a randomly inhomogeneous medium bounded by an ideally conducting plane. The mean square values of the fluctuations of phase and amplitude at a variable receiving point are found. Polarization effects and amplitude and phase correlations at two different receiving points are discussed. {The translation is mediocre.}

R. A. Silverman (New York)

503:

Koo, Benjamin Y.-C.; Katzin, Martin. An exact earth-flattening procedure in propagation around a sphere. J. Res. Nat. Bur. Standards Sect. D **64D** (1960), 61–64.

The radio-wave propagation through a stratified atmosphere above the spherical earth can be approximated by that above a plane earth, provided that the profile of the atmospheric refractive index is replaced by a corresponding, properly chosen, profile of another modified refractive index N. In order to investigate the corrections associated with this approximation the authors introduce the new variables $\eta = a \log \{(a+h)/a\}$ and $\tau = a\varphi$ instead of the height h above the earth and the angle φ measuring horizontal distances (a = earth's radius). A differential equation is then obtained for a function of η, and another for a function of φ, if these functions have to constitute the factors of those solutions of the wave equation in which η and φ enter separated. The differential equation in η (height-gain differential equation) has the same form as in the conventional theory, apart from the different profile for the relevant refractive index. However, the differential equation for τ now differs from Legendre's equation (holding for the variable φ). The authors consider this equation for the special case of a standard atmosphere defined by a linear dependence of N^2 on η; the equation can then be solved with the aid of an expansion with respect to a^{-2}, each term of which proves to be proportional to a higher-order Hankel function. It is thus possible to get corrections to the conventional theory for such a standard atmosphere. *H. Bremmer* (Eindhoven)

504:

Mével, Jean. Contribution à l'étude de la diffraction des ondes électromagnétiques par les sphères. Ann. Physique (13) **5** (1960), 265–320.

505:

Brysk, H. The radar cross section of a semi-infinite body. Canad. J. Phys. **38** (1960), 48–56.

Author's summary: "The concept of cross section as applied to a semi-infinite scattering body seems to require some clarification. The need for careful formulation of the problem arises because of the simultaneous occurrence of two characteristic lengths tending to infinity; the range from the radar to the target, and the size of the target. The infinite range assumption in the definition of the cross section allows the incident wave to be approximated as a plane wave in the case of a finite scatterer. For a semi-infinite body, it is customary to retain the plane-wave incidence, and introduce ad hoc arguments to dispose of the awkwardness due to the infinite extent of the scatterer. A return to the basic definition of a cross section, and examination of its motivation, lead here to an unequivocal formulation for the cross section of a semi-infinite body. Its consequences are pursued in the physical optics approximation. In particular, the nose-on backscattering from a body of revolution is exhibited, and results are computed for the paraboloid and the cone (which turn out to agree with the traditional ones). The broadside backscattering from a cylinder is also calculated, and the difference in this case between monostatic backscattering and the return in the backward direction from an incident plane wave is discussed."

K. C. Westfold (Sydney)

506:

Braunbek, Werner. Zur Beugung an Öffnungen in nichtebenen Schirmen. Z. Physik **156** (1959), 66–77.

This article deals in particular with the diffraction by a semi-infinite conical screen that is truncated by a circular aperture; the incident wave is assumed as a plane wave propagating in the direction of the axis of the cone. A

correction to Kirchhoff's approximation is evaluated, at points on the axis, with the aid of Kirchhoff's integral. In this integral the wave function and its normal derivative (insofar as not given by the boundary conditions) across the shadow side of the conical screen and the aperture are approximated by Sommerfeld expressions; the latter correspond to a half plane tangenting the cone along its intersection with the meridional plane through the point under consideration. The necessary integrations of the error functions entering in the Sommerfeld expressions lead to simple trigonometric formulas containing no integrals whatever. This applies both to the scalar problem (for a vanishing value of the wave function or of its normal derivative along the screen) and to the vectorial electromagnetic problem for a perfectly conducting cone. The results reduce to expressions derived earlier in the limiting case of a plane circular aperture.

H. Bremmer (Eindhoven)

507:

Jovi, Maria Grazia. Sulla instabilità dei modi dissimmetrici in una guida a sezione circolare. Atti Accad. Sci. Ist. Bologna. Cl. Sci. Fis. Rend. (11) **6** (1958/59), no. 2, 91–95.

508:

Govorun, N. N. Integral equations for an antenna in the form of a body of revolution with an impedance surface. Dokl. Akad. Nauk SSSR **126** (1959), 49–52 (Russian); translated as Soviet Physics. Dokl. **4**, 562–565.

An integral equation is obtained for the current distribution on the surface of an antenna whose surface impedance is anisotropic, and excited by an arbitrary incident field. The equation is essentially a modification of the equation of G. E. Albert and J. L. Synge [Quart. Appl. Math. **6** (1948), 117–131; MR **10**, 342; see p. 124]. The solution of the equation is not treated.

G. Sinclair (Toronto)

509:

Gabarre, Leandro; Cairó, Lorenzo. Méthode variationnelle pour la propagation des ondes électromagnétiques dans un plasma. C. R. Acad. Sci. Paris **249** (1959), 1750–1752.

The perturbation produced by a thin layer of gyroelectric plasma on the propagation constant of a rectangular waveguide which propagates only the dominant TE_{10} mode is calculated using a variational principle derived by Hauser [Quart. Appl. Math. **16** (1958), 259–272; MR **21** #3201]. The calculation is exactly analogous to that carried out by Hauser for an infinite gyromagnetic slab.

R. D. Kodis (Providence, R.I.)

510:

Amstutz, Pierre. Sur un problème d'approximation qui se présente dans le calcul des réseaux. Ann. Télécommun. **14** (1959), 111–121.

Dans la théorie des filtres électriques, le problème de l'optimisation au sens de Tchebycheff conduit à trouver une fonction rationnelle dont le carré a tous ses maxima égaux et tous ses minima non nuls égaux. La théorie des fonctions elliptiques et hyperelliptiques permet d'apporter une solution analytique. L'auteur expose divers procédés permettant le calcul de ces fonctions sans recourir aux tables existantes dont l'emploi est peu commode et diverses approximations valables dans des cas particuliers.

J. Kuntzmann (Grenoble)

511:

Crandall, Stephen H.; Siebert, William H.; Hoquetis, Bernard P. The response of linear system to non-Gaussian random inputs. J. Aero/Space Sci. **27** (1960), 154–155.

This note presents a counter-example to a result of Mazelsky [J. Aero. Sci. **21** (1954), 145–153; MR **15**, 545] who showed that if a wide band non-Gaussian stationary random process is impressed on a linear system with a narrow band transfer function, the response is a stationary random function with a probability distribution that is asymptotically Gaussian as the ratio of input bandwidth to the system transfer bandwidth approaches infinity. Reviewer believes that this counter-example demonstrates at least the necessity for additional convergence conditions on the integral of the input correlation function if Mazelsky's result is to be valid.

O. M. Phillips (Baltimore, Md.)

512:

Haus, H. A. Network theoretical and physical limitations of amplifier noise performance. Nuovo Cimento (10) **13** (1959), supplemento, 416–429.

This paper reports the results of the investigation of lossless network transformations on the noise figure of two terminal-pair high-gain amplifiers. In a previous report by the author it was shown that every two terminal-pair network has two invariants with respect to lossless transformations. This leads to a canonical form (at a given frequency) for two terminal-pair networks consisting of two separate resistances in series with uncorrelated noise voltage generators. Assuming that a network is found by means of which the amplifier can be represented in its canonical form, it is then shown that optimum noise performance for amplifier interconnections is realized when the interconnecting network is a lossless, non-reciprocal four-port device called a "circulator". The properties of the circulator are specified in terms of input-output relations between ports.

R. Kahal (Washington, D.C.)

CLASSICAL THERMODYNAMICS, HEAT TRANSFER

See also 444, 445.

513:

Kučerov, R. Ya.; Rikenglaz, L. Ê. On hydrodynamic boundary conditions for evaporation and condensation. Ž. Èksper. Teoret. Fiz. **37** (1959), 125–126 (Russian); translated as Soviet Physics. JETP **10** (1960), 88–89.

Authors' summary: "The boundary conditions have been found for hydrodynamic equations in the presence of evaporation and condensation. For small evaporation rates the temperature jump and the deviation of the vapor pressure from the equilibrium value are shown to be of the order of the ratio between the speed of the vapor flow v and the mean speed of heat transfer c. It is shown that the expressions commonly used in the literature for the flow of materials and heat in the presence of evaporation and condensation contain an error."

514:

Fujii, Tetsu. An analysis of turbulent free convection heat transfer from a vertical surface. Bull. JSME **2** (1959), 559–563.

515:

Cooper, D. C.; Hoskin, N. E. Thermal waves in irradiated graphite. Quart. J. Mech. Appl. Math. **12** (1959), 393–406.

Authors' summary: "This paper presents the mathematical formulation of a problem in the release of Wigner energy in a graphite bar of variable specific heat and thermal conductivity. The results of six such illustrative computations, obtained with the help of the I.B.M. 704 computer, are given to show the formation and motion of a thermal wave down the graphite bar."

516:

Sevruk, I. G. Transient heat convection within a spherical film. J. Appl. Math. Mech. **22** (1958), 587–593 (419–423 Prikl. Mat. Meh.).

The author considers the problem of transient heat conduction and convection in a viscous fluid confined within the space bounded by two concentric spheres, the fluid being originally at rest and the whole system being at a uniform initial temperature. The motion is generated by a step change in temperature of the spherical walls. Under the assumption of weak convection the problem is solved by linearization in terms of a series expansion in the Rayleigh number; the solution of the zeroth and first approximations are obtained.

T. Yao-tsu Wu (Pasadena, Calif.)

517:

Ioffe, I. A. Plane nonstationary heat conduction problem for a semi-infinite body with an internal isothermal cylindrical source of heat. Ž. Tehn. Fiz. **29** (1959), 417–422 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 369–374.

The system in question consists of an infinitely long circular cylinder which is completely immersed in a homogeneous semi-infinite body, the whole system being at a uniform initial temperature. The transient heat conduction in the semi-infinite body resulting from a step change in the surface temperature of the cylinder, with the temperature on the bounding plane kept fixed at its initial value, is considered for the case of large depth of immersion so that the temperature condition on the cylindrical surface is satisfied only approximately. The temperature field and the heat flux are then evaluated and discussed.

T. Yao-tsu Wu (Pasadena, Calif.)

QUANTUM MECHANICS

See also 416, 417, 604.

518:

Février, Paulette. Logical structure of physical theories. The axiomatic method. With special reference to geometry and physics. Proceedings of an International Symposium held at the Univ. of Calif., Berkeley, Dec. 26, 1957–Jan. 4, 1958 (edited by L. Henkin, P. Suppes and A. Tarski), pp. 376–389. Studies in Logic and the Foundations of Mathematics. North-Holland Publishing Co., Amsterdam, 1959. xi+488 pp. $12.00.

The paper is introduced by a discussion of three desiderata of physical theories: adequacy (predictive power); possession of structure (axiomatisation); ascription of physical or intuitive meaning to the terms of the theory. It is not too clear from the context what is meant by the last-named: the author refers vaguely to "something like 'Weltanschauung' " (pp. 378, 379).

The "physico-logical" approach here adopted keeps in view all three aspects. Corresponding to the "sharp" observables of classical mechanics, a calculus for experimental sentences is built up; the corresponding calculus of "propositionals" (a propositional being a strong logical sum of experimental sentences), taking care of "the lack of precision" of quantum mechanical measurements, follows, leaning on the work of Birkhoff and von Neumann, and more generally, that of Destouches. The second calculus is not a boolean algebra but an algebra of projective geometry.

An example is given of the opposition between the so-called classical probabilistic interpretation of wave mechanics, and Bohm's and de Broglie's causal and deterministic interpretation. The author purports to show that using the formal calculuses here mentioned one can prove that it is possible to pass from the classical quantum theory to a causal theory, and vice versa.

The author concludes that the price to pay for a refusal to accept indeterminism is that one has to work with certain physical entities which escape experimentation. But this is a matter for choice. With respect to the first aspect of theories (adequacy) they are equivalent.

{Stylistically, this paper could have been improved by a more adequate translation into English; the proof-reading also leaves much to be desired.}

G. Buchdahl (Cambridge, England)

519:

Aeschlimann, Florence. Le barycentre en théorie fonctionnelle des systèmes de corpuscules. J. Phys. Radium **21** (1960), 115–120. (English summary)

Author's summary: "Definition of the barycentric wave for a system of n particles in the functional theory, in the relativistic case and in the non-relativistic case with spin and isospin or without spin. Equation for the barycentric wave in the relativistic case and in the non-relativistic case. Case of particles of the same mass. Motion around the barycentre; equations for the relative motion. Particular case of a two particle system."

520:

Destouches, Jean-Louis. Sur une propriété limite de l'onde moyenne en théorie fonctionnelle des systèmes de corpuscules. J. Phys. Radium **21** (1960), 145–148. (English summary)

Author's summary: "It is shown that under four conditions, the mean wave for a great number of particles in the non-linear functional theory of particles obeys a linear equation with a probability very near unity. This equation is the same as the wave equation in the usual wave mechanics for the corresponding problem. Thus the linear part of the equation for the u-wave in the functional theory of particles is determined."

521:
Yêm, Pham Xuân. Interactions entre matière et rayonnement en théorie fonctionnelle. J. Phys. Radium **21** (1960), 185–188. (English summary)

Author's summary: "The interactions between an electron and many photons are expressed by the terms of coupling in the equations of those two types of particles: the action of the electron is represented in the electromagnetic equations of the photons in terms of electric and magnetic moments and in charge and current densities. Conversely, the potentials created by the photons will react upon the movement of the electron."

522:
Datzeff, Assène. Sur l'interprétation de la Mécanique quantique. Cas de présence d'un champ électromagnétique. C. R. Acad. Sci. Paris **249** (1959), 2725–2727.

523:
Datzeff, Assène. Sur l'interprétation de la Mécanique quantique. Cas de vitesse relativiste d'un corpuscule sans spin. C. R. Acad. Sci. Paris **250** (1960), 67–69.

An attempt to derive the Klein-Gordon equation based on correspondence principle arguments and the requirement that the Schrödinger equation be the limiting equation in the non-relativistic limit. The article contains remarks concerning the *WKB* method when applied to the Klein-Gordon equation.

S. Schweber (Waltham, Mass.)

524:
Englert, F. Renormalisation de masse d'un électron dans un corps noir. Acad. Roy. Belg. Bull. Cl. Sci. (5) **45** (1959), 782–789.

The self-energy of an electron in a hohlraum of temperature T is computed to second order in perturbation theory. It is shown that the effect of the interaction with the real photons of the hohlraum (to order α) is to increase the electron mass by δm, where $\delta m = (\alpha/12)(kT/mc^2)^2 m$.

S. Schweber (Waltham, Mass.)

525:
Martin, J. L. Generalized classical dynamics, and the "classical analogue" of a Fermi oscillator. Proc. Roy. Soc. London. Ser. A **251** (1959), 536–542.

This paper was written partly to elucidate the formulation of the Feynman principle for a Fermi system (which demands the use of anti-commuting eigenvalues). [See also the following review.] By generalizing the definition of a Poisson bracket, Hamiltonian dynamics can be extended to systems in which coordinates and momenta are not arranged in conjugate pairs. The total number of dynamical variables may even be odd. Such a generalized system will not usually have a Lagrangian. The new Poisson brackets allow one to formulate (classical) Hamiltonian dynamics for a system whose variables are anti-commuting quantities—a case which turns out to have a Lagrangian.

The problem of quantizing such generalized systems is discussed, particularly the example of a single Fermi oscillator.

J. C. Taylor (Rochester, N.Y.)

526:
Martin, J. L. The Feynman principle for a Fermi system. Proc. Roy. Soc. London. Ser. A **251** (1959), 543–549.

Here is considered the vexing question of how properly to treat the anti-commuting eigenvalues which appear when one attempts to apply the Feynman principle to a Fermi system. [See the preceding review.] The main tool is a well-defined but highly abstract symbol S_λ which takes the place of the sum over eigenvalues in the completeness equation

$$S_\lambda |\lambda)(\lambda| = 1.$$

The "sum over histories" which appears in the final formulation of the Feynman principle is to be understood as an infinite repetition of such S-operations. With this refinement, it is concluded that the Feynman principle for Fermi systems can be consistently formulated.

J. C. Taylor (Rochester, N.Y.)

527:
Loinger, A. Sulla questione ergodica della teoria quantistica. Rend. Sem. Mat. Fis. Milano **29** (1959), 3–12. (English summary)

Author's summary: "After having briefly recalled origin and developments of the ergodic question in classical and quantum mechanics, some results of recent researches, made in collaboration with P. Bocchieri, regarding the ergodic theorem of quantum mechanics are illustrated. In this work it is proved that the averaging procedure on the basis of the 'energy stratum' introduced by V. Neumann is alone sufficient to justify, in each instant, the use of the microcanonical quantum system, while the time-evolution of the system is completely irrelevant. It may then be concluded that the ergodic quantum problem will have to be approached in a completely new way."

528:
Hagedorn, R. Note on symmetry operations in quantum mechanics. Nuovo Cimento (10) **12** (1959), supplemento, 73–86.

Following an introduction in which the notions of completeness of a set of quantum mechanical variables and superselection rules are defined, a formal definition of a symmetry group—a group under which a physical system is invariant—is given. The body of the paper consists of proofs of the consequences which follow from the existence of a symmetry group. In particular, the author gives a simpler proof of a theorem proven earlier by Wigner [*Gruppentheorie und ihre Anwendung auf die Quantenmechanik, der Atomspektren*, Vieweg, Brunswick, 1931; note p. 251] that the symmetries of quantum mechanical systems give rise to either unitary or anti-unitary representations.

L. C. Maximon (Washington, D.C.)

529:
Novozhilov, Yu. On the construction of the S matrix for dressed particles. Nuclear Phys. **15** (1960), 469–479.

Author's summary: "The problem of setting up the S matrix for dressed particles is considered by representing the unperturbed states of the dressed particles by means of asymptotic states and by assuming an exponential structure of the vacuum state. Auxiliary independent

fields are introduced to describe dressed particles and the scattering theory is formulated in terms of operators and state vectors in the auxiliary field space. The energy operator in the auxiliary field space (effective Hamiltonian) is divided into two parts: the 'free' Hamiltonian, whose eigenstates include states of non-interacting dressed particles, and the interaction operator which specifies the particle interaction (but not interaction between the fields)."

530:

Hilgevoord, J. On the reduction formulae for the S-matrix elements. Nuclear Phys. **15** (1960), 657–663.

Author's summary: "Reduction formulae for the S-matrix elements, given by Lehmann, Symanzik and Zimmermann, are derived with particular emphasis on a proper use of the asymptotic condition and on the order of integration which is not always irrelevant. It is also stressed that local commutativity is not needed for the derivation of most of these formulae."

531:

Konisi, Gaku; Ogimoto, Takesi. On Levinson's theorem in the theory of multi-channel scattering. Progr. Theoret. Phys. **22** (1959), 807–813.

In 1949 N. Levinson [Dan. Vid. Selsk. Mat. Fys. Medd. **25** (1949), no. 9; MR **10**, 710] proved that there exists a relation between the difference of the scattering phase shift at infinite energy and the corresponding phase shift at zero energy on one side and the number of bound states of the system on the other side for a non-relativistic particle scattered by an ordinary potential. More recently, M. Ida has demonstrated that there exists a similar but not identical relation for a particular model of a field theory [Progr. Theoret. Phys. **21** (1959), 625–639; MR **21** #6222]. In this model one has one kind of relativistic mesons (θ-particles) which are scattered by heavy particles (N-particles) and the interaction goes via one or more heavy particles (V-particles). Ida's result reads

$$\delta(\infty)-\delta(\mu) = (m_0-m)\pi,$$

where $\delta(E)$ is the scattering phase shift at energy E, μ the mass of the meson, m the number of stable V-particle states (these correspond to the bound states in non-relativistic scattering) and m_0 the number of bare V-particle states in the model. In the present paper, the authors discuss a more general model where multi-channel scattering is also allowed. They consider one kind of N-particles, two kinds of V-particles and two kinds of θ-particles. The interaction is such that all reactions of the type $V_\lambda \rightleftarrows N+\theta_\alpha$ is allowed for all combinations of the indices λ and α. The particular scattering process $N+\theta_\beta \to N+\theta_\alpha$ is considered in detail. Because of the possibility of inelastic processes ($\alpha \neq \beta$) the diagonal phase shifts ($\alpha=\beta$)δ_α are, in general, complex quantities. The authors show that the above relation of Ida's is here generalized to

$$\mathrm{Re}\,[\delta_\alpha(\infty)-\delta_\alpha(\mu_\alpha)]+\arg\frac{\sin\delta_\alpha(\infty)}{\sin\delta_\alpha(\mu_\beta)} = (N_\alpha-\bar{N}_\alpha)\pi,$$

where $\bar{N}_\alpha$ is the number of stable V_α particles (bound states of the $N-\theta_\alpha$ system) while N_α is an integer which describes the number of zeros of a certain auxilliary function. The authors stress that this number N_α is not directly related to the number of bare V-particles in the model. *G. Källén* (Lund)

532:

Čičerin, A. G. Use of scattering amplitude to reconstruct the potential near its boundary. Ž. Èksper. Teoret. Fiz. **36** (1959), 1750–1757 (Russian); translated as Soviet Physics. JETP **9**, 1248–1252.

Author's summary: "The 'asymptotic Born approximation' is formulated from the scattering amplitude for a potential that coincides with the true potential at sufficiently large radii. The order of magnitude of the relative error in the determination of the scattering potential at a point r is $\int_r^\infty U(r')r'dr'$. The case where the scattering amplitude is given in a finite energy interval is considered."

533:

Gottfried, K. Dispersive effects in elastic electron scattering by complex nuclei. Nuclear Phys. **15** (1960), 92–101.

Author's summary: "A Z-independent estimate of the validity of the conventional analysis of elastic electron scattering is made with the help of an optical potential. It is concluded that the dispersive corrections are always of order 1/137." *John G. Taylor* (Cambridge, England)

534:

Ichikawa, Yoshi H.; Yamamoto, Masaaki. Bremsstrahlung in high density media at high energies. Progr. Theoret. Phys. **23** (1960), 81–86.

Authors' summary: "Qualitative discussions are presented concerning the effects of the collective motion of media for the bremsstrahlung process in high density at high energies. The derived formula of the bremsstrahlung cross section has similar structure as the Nozieres-Pines' cross section of Compton scattering obtained by taking into account effects of the collective motion of electrons." *L. C. Maximon* (Washington, D.C.)

535:

Zaidi, M. H. Double scattering of electrons with a dipole moment. Phys. Rev. (2) **116** (1959), 241–243.

Author's summary: "The double scattering of electrons with a small intrinsic electric dipole moment is investigated theoretically for the case where magnetic and electric fields are present in the space between the two targets. It is shown by using density matrix techniques that the plane of polarization of the electron beam, after the first scattering, would be rotated by the magnetic and electric fields just as predicted by regarding the electrons as classical spinning tops with magnetic moments and classical electric dipoles precessing about the directions of the magnetic and electric fields, respectively."

536:

Weinmann, A. Bounds for elastic and inelastic scattering. Proc. Phys. Soc. **75** (1960), 510–519.

Author's summary: "Two methods are given for obtaining bounds for functions satisfying certain systems of coupled integral equations. Application to the integral equations for potential scattering (elastic or inelastic,

three dimensional or in partial wave expansion) results in bounds for the wave functions, and hence for the scattering amplitudes and cross sections. The method gives sufficient conditions for the convergence of the Born iteration series, and estimates for the truncation errors for a wide variety of problems. A simple example illustrates the methods. It is also indicated how they can be employed in more complicated situations than those considered here."

537:

Deo, B. On fermion loops of two vertices. Indian J. Phys. **34** (1960), 159–168.

Author's summary: "The imaginary part of the retarded matrix element for a closed loop of two vertices has been deduced by perturbation theory. It has been used to evaluate the photon and meson vacuum polarisation effects by assuming dispersion relations. Some difficulties regarding the mesic vacuum polarisation have been shown to be removed by considering the vertex correction.

The formulae have been applied to deduce the decay rates of some fundamental particles and the results obtained are in good agreement with experiments."

538:

Gardner, J. W. Cascade theories and the Landau approximation. Proc. Phys. Soc. **75** (1960), 205–216.

A variation-iteration method is used to find the angular distribution, integrated over all depths, of the electrons in an electron-photon cascade. The scattering of electrons is represented by the screened Coulomb cross section, in Born approximation, and in the small-angle approximation; pair production and Bremsstrahlung are approximated by "super-simplified" cross sections; collision loss is ignored; i.e., the calculation applies to electrons well above the critical energy. The main interest lies in comparison with earlier work, in which the integral operator for electron scattering has been replaced by a second-order differential operator, multiplied by the second moment of the scattering kernel (Landau approximation). The error due to this approximation is nearly a factor 2 at small angles; this large error is due to the long "tail" of the screened Coulomb scattering law. By contrast, the error due to the use of super-simplified cross sections is only about 12% at small angles. *J. M. Blatt* (Sydney)

539:

Sitenko, A. G. Interaction of deuterons with nuclei. Uspehi Fiz. Nauk **67** (1959), 377–444 (Russian); translated as Soviet Physics. Uspekhi **2**, 195–235.

In the interval since the appearance of this paper there have been significant enough changes in our understanding of various aspects of the interaction of deuterons with nuclei to make this paper quite out-of-date, particularly with respect to the low and medium energy interactions. On the other hand the author has presented a fairly unified and complete description of the theoretical formulations of the interaction of deuterons with nuclei.

The paper is an excellent review of a limited field without any attempts to justify or generalize the conclusions of the various calculations.

N. S. Wall (Cambridge, Mass.)

540:

Frantz, Lee M.; Mills, Robert L. Many-body basis for the optical model. Nuclear Phys. **15** (1960), 16–32.

It is shown that the scattering of a nucleon by a closed shell (or closed shell ± 1) nucleus can be described exactly in terms of a complex, non-local, energy dependent, and angular momentum dependent optical model potential. A formal expression is given for the optical model potential in terms of the two nucleon interaction. The proof is based on an analysis of the Goldstone formula in terms of graphs.

One point concerning the non-locality which is emphasized is that the optical potential contains a projection operator off the occupied model states. The closing sentences of the paper are devoted to a discussion of the well known question whether one should do anything about this point in phenomenological calculations with local optical model potentials.

J. L. Gammel (Los Alamos, N.M.)

541:

Rodberg, Leonard S. The optical model and inelastic scattering. Ann. Physics **9** (1960), 373–390.

This paper provides a foundation for the results of Levinson and Bannerjee, who applied the direct interaction model of inelastic scattering to the $C^{12}(p, p')C^{12}$ data. Levinson and Bannerjee found that the optical potential seen by the scattered particle is weaker than for the elastic scattering. In this paper, it is shown that the "$1/A$ corrections" to the inelastic scattering amplitude, which arise from the requirement that the contribution of a given target nucleon to the optical potential be removed when that nucleon interacts directly with the incident particle, explain the reduction. Levinson and Bannerjee also found that the effective two body interaction in the direct process is very strong, but it is pointed out that this is in agreement with other estimates of the "t-matrix" in nuclear matter.

J. L. Gammel (Los Alamos, N.M.)

542:

Rodberg, Leonard S. Relation between angular distribution and polarization in the optical model. Nuclear Phys. **15** (1960), 72–78.

A condition under which the spin polarization of a nucleon scattered by a nucleus is proportional to the derivative of the differential cross section is derived. The condition is derived assuming (1) the validity of the optical model and that (2) the spin orbit term is weak compared to the central term. The condition is that the matrix elements of the spin orbit term calculated between wave functions determined including only the central term be independent of the orbital angular momentum (l). It is stated that this is a reasonable requirement on a potential model, which favours no particular state and implies a smooth variation with l.

As a by product an estimate of the depth of the central term from the position of the minima in the differential cross section is derived. An approximate expression for the depth of the spin orbit term in terms of measured polarizations is also derived.

J. L. Gammel (Los Alamos, N.M.)

543:

Tani, Smio. Scattering involving a bound state. Phys. Rev. (2) **117** (1960), 252–260.

This paper considers potential scattering when there is the possibility of a bound state being formed. It is assumed that the bound state is formed in a time that is short compared to that involved in the scattering process. Creation and annihilation operators may then be introduced to describe the bound state as an independent entity, redundancy being prevented by a subsidiary condition on state vectors. This condition implies that only intermediate states orthogonal to the bound state appear in a multiple scattering process. It is possible by this method to give a non-rigorous explanation of Levinson's theorem [Danske Vid. Selsk. Mat.-Fys. Medd. **25** (1949), no. 9; MR **10**, 710] that $\delta(0)-\delta(\infty)=\pi$ in the case of one bound state, where $\delta(k)$ is the S-wave phase shift at momentum k. The jump of $\delta(0)$ by π when the bound state appears is in that part of the phase shift arising from scattering by the projection of the kinetic energy in states orthogonal to the bound state. The remainder of the scattering is by an effective non-local potential to which usual scattering theory is conjectured to be applicable. *John G. Taylor* (Cambridge, England)

544:

Grawert, Gerald; Petzold, Joachim. Formale Mehrkanal-Streutheorie. Eine Alternativ-Formulierung. Z. Naturf. **15a** (1960), 311–319. (English summary)

Authors' summary: "An alternative formulation is presented of the formal theory of multi-channel scattering in non-relativistic quantum mechanics. We start by defining spaces of state vectors, where two particles either stay together or separate in the limit $t\to+\infty$ (or $-\infty$), when the state vector develops in time by e^{-iHt} (H is the complete Hamiltonian of the n-particle system). A channel is defined as a space of state vectors with the following property: Developing in time by e^{-iHt} they asymptotically describe a state of the n-particle system, where the particles are grouped in fragments. Defining a Hamiltonian H_γ for each channel, in which—compared to H—the interactions acting between particles from different fragments are missing, it is physically plausible that $\lim e^{iH_\gamma t}e^{-iHt}\Psi$ exists for vectors Ψ in the channel. Having discussed the limit vectors (asymptotic states), the S-matrix formalism can be introduced as usual. Finally the introduction of the exclusion principle is discussed."

545:

Zinnes, I. I. Two theorems on scattering. Nuovo Cimento (10) **12** (1959), supplemento, 87–99.

The following two theorems are proved. (I) Let ψ be a vector in the Hilbert space $\mathscr{H}$ of realizable states of a system consisting of a single spinless particle. Let Δ be an arbitrary Borel set in Euclidean three space E_3 and let E_Δ be a projection operator with the property:

$$(E_\Delta f)(x) = f(x) \quad \text{if } x\in\Delta,$$
$$= 0 \quad \text{if } x\notin\Delta,$$

where $f(x)$ is an L^2 function defined in E_3. Let $U_t=\exp(-iH_0t)$ with $H_0=\mathbf{p}^2/2m$; then $\lim\|E_\Delta U_t\psi\|\to 0$ as $t\to\pm\infty$. This theorem is essentially a slight generalization of a theorem of Cook [J. Math. Phys. **36** (1957), 82–87; MR **19**, 1011].

(II) Let $\{U_t^{(\alpha)}\}$ and $\{\overline{U}_t^{(\alpha)}\}$ ($\alpha=1, 2, \cdots$) be two sets of unitary groups such that

$$U_t^{(\alpha)} = \exp[-iH_\alpha t],$$
$$\overline{U}_t^{(\alpha)} = \exp[-i\overline{H}_\alpha t].$$

It is explicitly assumed that H_α and $\overline{H}_\alpha$ have the form

$$H_\alpha = \sum_{j=1}^{n} a_j^{(\alpha)}\mathbf{p}_j^2,$$

$$\overline{H}_\alpha = \sum_{j=1}^{n} \bar{a}_j^{(\alpha)}\mathbf{p}_j^2,$$

where $\{a_j^{(\alpha)}\}$ and $\{\bar{a}_j^{(\alpha)}\}$ are sets of real numbers. Let $V_t=\exp[-iHt]$ be a unitary group such that for each α, $\lim_{t\to\pm\infty} V_t^*U_t^{(\alpha)}$ and $V_t^*\overline{U}_t^{(\alpha)}$ exist on closed domains $\mathscr{D}_\alpha$ and $\overline{\mathscr{D}}_\alpha$. These limits respectively define the Møller scattering operators $\Omega_\pm^{(\alpha)}$, $\overline{\Omega}_\pm^{(\alpha)}$ which are partial isometries with ranges $R_\pm^{(\alpha)}$ and $\overline{R}_\pm^{(\alpha)}$ respectively. Assume that $R_+=\overline{R}_+=R_-=\overline{R}_-$; then the sets $\{H_\alpha\}$ and $\{\overline{H}_\alpha\}$ are identical except possibly for the order of their elements.

S. Schweber (Waltham, Mass.)

546:

Ozaki, Shoji. On the consistency of quantization in quantum electrodynamics. Nuclear Phys. **15** (1960), 501–509.

Author's summary: "It is shown that the new relativistically covariant quantization method of quantum electrodynamics in the Heisenberg representation leads to a self-consistent and definite formalism. If we divide the electromagnetic 4-potentials into a transverse part, a part related to Coulomb potential, and the remaining indefinite part, which is needed to satisfy identically the fundamental equations when the Lorentz condition is not imposed, the new relativistically covariant quantization determines the commutation relations of the indefinite potential. By transforming the formalism of quantum electrodynamics obtained by the method mentioned above in Heisenberg representation into the interaction representation and determining the interaction Hamiltonian by this procedure, the equivalence to the usual quantum electrodynamics can be proved without any indefiniteness."

547:

Redmond, Peter J.; Uretsky, Jack L. In-out formalism for nonrelativistic quantum mechanics. Ann. Physics **9** (1960), 106–124.

This article contains a review of the second quantized description of a system of nonrelativistic particles interacting through two-body forces in terms of Heisenberg operators $\psi(\mathbf{x}, t)$ obeying the equation of motion

$$i\partial_t\psi(\mathbf{x}, t)+\nabla^2\psi(\mathbf{x}, t) = \int d\mathbf{y}\psi(\mathbf{y}, t)V(\mathbf{x}-\mathbf{y})\psi(\mathbf{y}, t)\psi(\mathbf{x}, t)$$

and the equal time commutation rules

$$[\psi(\mathbf{x}, t), \psi^*(\mathbf{y}, t)]_\pm = \delta(\mathbf{x}-\mathbf{y})$$

(+for a system of Fermions, − for a system of Bosons). It is shown that if $\{f_\alpha(\mathbf{x}, t)\}$ is a normalizable solution of $\nabla^2 f_\alpha(\mathbf{x}, t)=i\partial_t f_\alpha(\mathbf{x}, t)$ with $\sum_\alpha f_\alpha(\mathbf{x}, t)\bar{f}_\alpha(\mathbf{x}', t)=\delta(\mathbf{x}-\mathbf{x}')$ then the operator

$$\psi_\alpha^*(t) = \int d\mathbf{x} f_\alpha(\mathbf{x}, t)\psi^*(\mathbf{x}, t)$$

converges weakly in the $\lim t \to +\infty$ and $\lim t \to -\infty$ to the operator $\psi_{\alpha\,\mathrm{out}}$ and $\psi_{\beta\,\mathrm{in}}$ respectively, if the potential $V(\mathbf{x}-\mathbf{y})$ vanishes for large $r=|\mathbf{x}-\mathbf{y}|$ faster than $r^{-(1+\varepsilon)}$ ($\varepsilon>0$). A similar conclusion is reached for the two-particle operator

$$\psi_2^*(t) = (2!)^{-1}\int d\mathbf{x}\int d\mathbf{y}\phi(\mathbf{x},\mathbf{y};t)\psi^*(\mathbf{x},t)\psi^*(\mathbf{y},t),$$

where $\phi(\mathbf{x},\mathbf{y};t)$ is a solution of the time-dependent Schrödinger equation for a two-particle system interacting through a potential $V(\mathbf{x}-\mathbf{y})$. A discussion of the case when ϕ corresponds to a bound state is included as well as the relation between $\psi^*_{2\,\mathrm{out}}[\psi^*_{2\,\mathrm{in}}]$ and $\psi^*_{\alpha\,\mathrm{out}}\psi^*_{\beta\,\mathrm{out}}$ $[\psi^*_{\alpha\,\mathrm{in}}\psi^*_{\beta\,\mathrm{in}}]$ in the case ϕ corresponds to a scattering state. The main contribution of the paper consists in a derivation of the commutation relations between the in and out operators. This derivation is based on a procedure due to Zimmermann [Nuovo Cimento (10) **10** (1958), 597–614; MR **21** #588], who formulated it for the relativistic situation. Reduction formulae are derived and the S matrix exhibited.

S. Schweber (Waltham, Mass.)

548:

Scarf, F. L. Anticommutator for a nonlinear field theory. Phys. Rev. (2) **115** (1959), 463–467.

Following Glaser [Nuovo Cimento (10) **9** (1958), 990–1006; MR **20** #6294], the operator $\psi(x)\psi^*(x')$ in Thirring's two-dimensional spinor field theory is written as an ordered functional of the free field operators ϕ and φ^*. Its renormalized vacuum expectation value is computed and considering the system placed in a box of length L (which introduces an infrared cut off), the result comes out

$$S_{12}'(x_1,t_1;x_2,t_2) = S_{12}(v_1-v_2)\exp\{[gg'/(2\pi)^2]\ln(L/|u_1-u_2|)\}$$
$$(u_1,u_2 \ll L),$$

where $v=x-t$, $u=x+t$, $g'=g+2\pi n$ so that $|g'/2\pi|<1$. This result differs strongly from those of the perturbation theory and the expansion over intermediate steps.

A. Raychaudhuri (Calcutta)

549:

Okun, L. B.; Rudik, A. P. On a method of finding singularities of Feynman graphs. Nuclear Phys. **15** (1960), 261–288.

Authors' summary: "The method of finding the singularities of Feynman graphs suggested by Landau is investigated. Techniques are evolved to establish whether any Feynman graph has a singularity. If it has, the method under discussion makes it possible easily to determine its location. To illustrate possibilities of the method the singularities of baryon form-factors and the nearest singularities of some simple scattering amplitudes are determined."

550:

Caianiello, E. R. Theory of coupled quantized fields. Nuovo Cimento (10) **14** (1959), supplemento, 177–191.

This paper is a report of lectures given at Varenna in 1958. It presents an account of the author's techniques for expressing the perturbation expansions of field theory in a neat algebraic form. Use is made of determinants and pfaffians for describing fermion propagators, and related quantities called permanents and hafnians for describing bosons. A definition of determinants and pfaffians in terms of Grassman algebras is given and their properties derived in a neat way.

With the aid of these algebraic quantities, and their expansion theorems, integral equations for the propagators of quantum electrodynamics can be derived from the perturbation expansion of the S-matrix.

C. A. Hurst (Adelaide)

551:

Haag, R.; Luzzatto, G. Differential equations for the renormalized fields in the point source Lee-model and scalar neutral meson theory. Nuovo Cimento (10) **13** (1959), 415–429. (Italian summary)

Authors' summary: "For two simple field theoretical models with infinite renormalization we show that the equations of motion can be formulated as differential equations for the renormalized fields involving only finite quantities. Thus the status of the initial value problem is not changed by the infinite renormalization. One has to observe that the field quantities at a sharp time, though not observables, have a well defined meaning as bilinear forms (infinite matrices). The equations involve certain limiting procedures which are closely related to Valatin's work on Quantum Electrodynamics but more explicit."

A. Klein (Philadelphia, Pa.)

552:

Gribov, V. N. The spectral representation of the two-meson Green's function. Soviet Physics. JETP **35** (8) (1959), 287–295 (416–427 Ž. Èksper. Teoret. Fiz.).

In this paper the causality conditions are used to obtain the spectral representation of the two-meson Green's function. From the result of the spectral representation the meson-meson scattering amplitude is shown to be an analytic function in the upper half of the complex ω-plane, and to have no essential singularity at infinity, where ω is the energy of one of the mesons in the center-of-mass system. As a result it is possible to obtain the dispersion relations for a fixed value of scattering angle in the center-of-mass system. Analytic continuation of these relations into the region of imaginary scattering angles is discussed.

T. Yao-tsu Wu (Pasadena, Calif.)

553:

Kobayakawa, Keizo; Imamura, Tsutomu. A field theoretical investigation of multiple meson production. I. Progr. Theoret. Phys. **23** (1960), 137–160.

A semi-field-theoretic treatment is given of multiple meson production, based on the assumption of a short collision time. The results are dominated by the usual covariant phase-space integrals, and change conservation is taken into account approximately. The results are then compared with experiment. It is alleged that the results differ from those of earlier formulations of the same problem.

H. W. Lewis (Madison, Wis.)

554:

Hara, O. Universality in the non-leptonic decay interaction of hyperons. Nuovo Cimento (10) **14** (1959), 114–118. (Italian summary)

Author's summary: "If we assume the $\Delta I=\frac{1}{2}$ rule and make some other plausible assumptions, decay interac-

tions for Λ and Σ can be determined except for the coupling constants g_Λ and g_Σ. g_Λ and g_Σ were determined from the life times of Λ and Σ, and it was found that they agree within the experimental error."

John G. Taylor (Cambridge, England)

555:

Taylor, J. C. Analytic properties of perturbation expansions. Phys. Rev. (2) **117** (1960), 261–265.

The analytic properties of Green's functions, in each order of perturbation theory, are discussed in this paper for real values of the invariants which may be formed from the external momenta. The values of the invariants chosen correspond to complex moments, since it is this "unphysical region" which may contain singularities not expected on physical grounds. The method used is to replace Minkowski space by a four-dimensional space S with a different signature, so that the invariants now arise from scalar products of real vectors in S. The Feynman integrals in perturbation theory may now be regarded as integrals over four-vectors in S, and their singularities obtained simply. This method is applied to derive a simple geometrical construction for the abnormal singularities in the vertex function and two-particle scattering amplitude. It is further applied to enlarge slightly the range of momentum transfer for which the fixed momentum transfer dispersion relations were proved by Symanzik in each order of perturbation theory [Progr. Theoret. Phys. **20** (1958), 690–702; MR **21** #1849], for nucleon-nucleon and pion-nucleon scattering. The method does not allow one to conclude that the limitations on momentum transfer derived by the author are final.

John G. Taylor (Cambridge, England)

556:

Abate, E.; Fabri, E. Use of an electronic computer for the construction of exact eigenfunctions of orbital angular momentum in L-S coupling. Nuovo Cimento (10) **14** (1959), 29–47. (Italian summary)

A discussion of the evaluation by means of an automatic computer of eigenfunctions of total orbital angular momentum for many-electron systems. Results are given for 3, 4, and 5 electrons on d-states and for 3 and 4 electrons in f-states, but neglecting electron spin.

D. F. Mayers (Oxford)

557:

Trefftz, E. Zur Relevanz des Energievergleiches als Probe für die Güte einer Näherungs-Wellenfunktion bei angeregten Zuständen. Z. Naturf. **14a** (1959), 708–712.

Author's summary: "Um die Güte einer genäherten Wellenfunktion zu beurteilen, wird im allemeinen der mit ihr gebildete Mittelwert der Energie mit dem experimentellen Termwert verglichen. Für angeregte Zustände ist dies nur sinnvoll, wenn die Orthogonalität der Wellenfunktion zu denen tieferer Zustände berücksichtigt wird. Für Wellenfunktionen die aus 1-Elektronen-Funktionen aufgebaut sind, wird angegeben, wie dies praktisch durchgeführt werden kann. Als Beispiel dienen Hartree-Fock- und Morse-Young-Haurwitz Funktionen für He."

D. F. Mayers (Oxford)

558:

Hall, G. G. Improved atomic wave functions using a functional transformation. Proc. Phys. Soc. **75** (1960), 575–581.

Author's summary: "The accuracy of an atomic wave function is often increased by an adjustment of the overall scale of the wave function. This method of improving a wave function is here generalized by considering a variable scale factor which can be an arbitrary function of the radius. This type of variation has the valuable property of leaving all the coefficients in the density matrices invariant so that it is much easier to minimize the energy than it is for most parameters. The new integrals which arise are relatively easy to evaluate.

To illustrate the power of the method an application is made to the simplest wave functions for the He isoelectronic sequence. A simple type of variation is found and the results show that a close approximation to the self-consistent-field energies can be obtained from a two-parameter wave function."

559:

Talmi, Igal. Symplectic invariance and the pairing property of nuclear interactions. Nuclear Phys. **16** (1960), 153–157.

Author's summary: "The condition of symplectic invariance of a general two-body interaction in the j^n configuration is derived for all values of T without the explicit use of group theory. The relation of symplectic invariance to the pairing property is also discussed."

560:

French, J. B. Symplectic symmetry in the nuclear shell model. Nuclear Phys. **15** (1960), 393–410.

Author's summary: "The nature of the general two-particle interaction which is compatible with symplectic symmetry in the jj coupling shell model is investigated. The essential result is that, to within an additive constant and an additive multiple of T^2, the interaction should have the form of a sum of scalar products of single-particle tensors which have odd rank in the single-particle j space. An example of an interaction satisfying these conditions is a central interaction with $\sigma_1 \cdot \sigma_2$ exchange nature. The condition for good symplectic symmetry is expressed also as a set of linear constraints on the two-particle energies, again as constraints on the particle-hole energies and finally in terms of the relationship between the particle-particle and particle-hole spectra. When we deal with identical particles only, the conditions for good symplectic symmetry (or seniority) are greatly relaxed and in particular are satisfied for a short-range (δ-function) interaction, as shown earlier by Racah and Talmi."

561:

Sawada, K.; Rockmore, R. M. Stability of nuclear matter. Phys. Rev. (2) **116** (1959), 1618–1619.

Authors' summary: "The stability condition for a model of collective excitations of nuclear matter due to Glassgold, Heckrotte, and Watson is derived; it is shown that it may be satisfied for the actual parameters (nuclear radius, strength of potential, etc.). It is possible that nuclear matter is stable in their approximation, although there is no compressional collective mode. The stability condition seems to imply values of η, the radius parameter used by Karplus and Watson, equal to or smaller than those values obtained from their simple saturating nuclear model."

562:

Moore, R. G., Jr. Nuclear reaction cross-section theory. Rev. Mod. Phys. **32** (1960), 101–116.

In effect this paper is a concise, up-to-date, summary of the theory of compound nuclear reactions as formulated by Weisskopf and his co-workers. Unfortunately the exclusion of direct interactions detracts from the utility of the paper for estimating actual reaction cross-sections, which seems to be a subsidiary aim of the paper.

N. S. Wall (Cambridge, Mass.)

563:

Newton, T. D. On the spherical oscillator nucleus. Canad. J. Phys. **37** (1959), 944–964.

In this paper some justification is sought for using the spherical oscillator wave function as the central potential of a single-particle model of a nucleus by examining the degree of self-consistency, in a quantitative fashion, of the oscillator model. The method is based on a Hartree-Fock calculation using a simple non-singular internucleon potential which correctly describes the two-nucleon interaction at low energies. In the calculation an effective mass for a nucleon in a nucleus equal to 0.757 times the mass of a free nucleon is used. The matrix elements between single-particle-occupied states of the Hartree-Fock Hamiltonian operator are computed and compared directly with the corresponding elements of the single-particle Hamiltonian. The comparison shows qualitatively that the oscillator wave functions are nearly self-consistent and the oscillator frequency derived is physically reasonable, but the bound on the binding energy is not good. It is also shown that the oscillator wave functions are a good approximation for the state functions of particles bound in a finite potential well having the shape of a cutoff oscillator so that the Hartree-Fock calculation can be used to prescribe a shell model potential.

T. Yao-tsu Wu (Pasadena, Calif.)

564:

Jarmain, W. R. Klein-Dunham potential energy functions in simplified analytical form. Canad. J. Phys. **38** (1960), 217–230.

Methods are given for calculating potential energy curves for diatomic molecules, except near the dissociation limit: these are compared with the Morse functions. Formulae are derived for the classical turning points in terms of vibrational quantum number, based on the Klein-Rees method. It is shown that the results are identical with the usual approximate form of the inverted Dunham series.

D. F. Mayers (Oxford)

565:

Lin, Chun C. On the interactions of nuclear spin with electrons in neighboring molecules. Ann. Physics **9** (1960), 325–331.

The interaction of a nuclear spin with the electrons in neighboring atoms and molecules is explored, and the relevant formulae are derived. It is found that the Fermi contact interaction remains valid. Applications to nuclear resonance experiments are discussed qualitatively.

H. W. Lewis (Madison, Wis.)

566:

Overend, John; Scherer, J. R. Transferability of Urey-Bradley force constants. I. Calculation of force constants on a digital computer. J. Chem. Phys. **32** (1960), 1289–1295.

Authors' summary: "An algorithm for the systematic calculation of Urey-Bradley force constants has been programed for a digital computer (the Datatron 204). The secular equation is set up and solved in internal coordinates, the potential energy being transformed from Urey-Bradley space to internal-coordinate space by a matrix Z. This same matrix is also used to transform the Jacobian of λ with respect to the force constants from internal-coordinate to Urey-Bradley space, thereby allowing the direct determination of Urey-Bradley force constants. A method is described whereby the Z matrix and Wilson's G matrix may be set up by the computer from the geometrical parameters of the molecule."

567:

Bingel, Werner A. Some properties of first order density matrices with special application to many-electron atoms. J. Chem. Phys. **32** (1960), 1522–1530.

Author's summary: "Properties of the first order density matrix $\gamma(\mathbf{x}'|\mathbf{x})$ of an N-electron system are discussed, including the relation between $\gamma(\mathbf{x}'|\mathbf{x})$ and the spinless first order density matrix $\gamma(\mathbf{r}'|\mathbf{r})$ and those symmetry properties of the total electronic charge density $\gamma(\mathbf{r}) \equiv \gamma(\mathbf{r}|\mathbf{r})$ which result from the corresponding properties of the total wave function with special reference to many-electron atoms. General formulas are given for the evaluation of $\gamma(\mathbf{r})$ of many-electron atoms in the orbital approximation and for its decomposition into terms of different angular symmetry. These formulas are applied to all terms of the electron configurations $s^n p^m$ and the results collected in tables."

568:

Koide, Shoichiro; Sinha, K. P.; Tanabe, Yukito. On the theory of superexchange interaction. Progr. Theoret. Phys. **22** (1959), 647–662.

The contribution to the superexchange interaction which arises from the deformation of the orbitals of the intervening ion is investigated quantitatively for the three-centre four-electron system. It is shown that this procedure of using different anion orbitals for electrons with opposite spins is equivalent to including certain excited configurations. The deformation leads to a splitting between the ferromagnetic and anti-ferromagnetic states provided that these are represented by single Slater determinants. If, however, singlet and triplet functions are used the splitting arises only from the effect of excited states in which one of the anion electrons changes its spin; this effect is estimated by perturbation theory. A rough numerical estimate gives reasonable agreement with the observed Neel temperature for MnO.

A. C. Hurley (Melbourne)

569:

Amati, D.; Vitale, B. Some topics regarding the strong interactions of strange particles. Fortschr. Physik **7** (1959), 375–421.

This is a useful review article, which analyses the experimental information available on the mass, isotopic spin and strangeness of strange particles, their relative parities, and the symmetry properties of strong inter-

actions. The discussion is mainly descriptive, and results of calculations only are given. *C. A. Hurst* (Adelaide)

570:

Granovskiĭ, Ya. I. On the calculation of the interaction constant in a nonlinear theory. Ž. Èksper. Teoret. Fiz. **37** (1959), 192–196 (Russian); translated as Soviet Physics. JETP **10** (1960), 135–137.

Author's summary: "The coupling constant is expressed in terms of the proton mass in the second approximation of the Tamm-Dancoff method. The formulas obtained can be used in the case in which the Lagrangian is an arbitrary linear combination of the five basic invariants."

571:

Thirring, W. Symmetry properties of elementary particles. Nuovo Cimento (10) **14** (1959), supplemento, 415–428.

These lecture notes give a clear and concise account of some fundamental symmetry properties of quantum field theory. The scalar and spinor representations of the homogeneous Lorentz group are defined. The essentials of quantization are introduced by way of a discussion of the Bose-Einstein and Fermi-Dirac oscillator, following the approach of Schwinger. This groundwork allows an immediate proof of the connection between spin and statistics and of the TCP theorem. The final sections summarize Thirring's own work on the minimum number of simple fields necessary to describe a system of particles with given "internal" symmetry properties, such as charge and isotopic spin. *J. C. Taylor* (Rochester, N.Y.)

572:

Ashkin, J. Strange particles. Nuovo Cimento (10) **14** (1959), supplemento, 310–327.

An elementary survey of the subject. The reasons for the classification scheme of Gell-Mann and Nishijima are clearly outlined. Effects of non-conservation of parity and methods of spin determination are described.

J. C. Taylor (Rochester, N.Y.)

573:

Sokolov, A. A.; Loskutov, Yu. M. Contribution to the theory of bosons and fermions with oriented spins. Ann. Physik (7) **5** (1959), 42–52.

Authors' summary: "A theory is developed for positive and negative energy charged bosons and fermions with spins directed oppositely with respect to the corresponding momenta. Invariance of the equations for a fixed spin direction is investigated."

574:

Łukaszuk, L. Note on symmetry of strong interactions. Nuclear Phys. **15** (1960), 513–515.

575:

Mandelstam, Stanley. Analytic properties of transition amplitudes in perturbation theory. Phys. Rev. (2) **115** (1959), 1741–1751.

This paper presents further work on the representation (double dispersion relation) for scattering amplitudes previously conjectured by Mandelstam [Phys. Rev. (2) **112** (1958), 1344–1360; MR **20** #5057]. First, the question of subtractions to the double dispersion relations is discussed. These may be either single dispersion integrals (functions of one variable only) or true constants. The kinds of subtraction are described which are expected to be necessary for different scattering processes. Second, it is proved that the representation correctly described fourth order perturbation theory, provided that the masses satisfy certain inequalities. This is done essentially by exhibiting the required spectral function and confirming that it yields the expected integral. It is mentioned that the proof can be extended to the sixth order. Third are described briefly the modifications that occur as the external masses are increased so that anomalous thresholds appear followed by singularities at complex points.

J. C. Taylor (Rochester, N.Y.)

576:

Mandelstam, Stanley. Construction of the perturbation series for transition amplitudes from their analyticity and unitarity properties. Phys. Rev. (2) **115** (1959), 1752–1762.

The double dispersion relation proposed by the author [Phys. Rev. (2) **112** (1958), 1344–1360; MR **20** #5057, and the preceding review] is used in conjunction with unitarity to generate certain terms of perturbation theory. First, fourth order perturbation theory is generated. To higher orders, production processes are neglected in the unitarity equations, to yield a set of perturbation terms which may be identified with the "one-meson" approximation. However, it is not possible to satisfy all the unitarity equations simultaneously in this approximation. It is shown how to determine the regions in which the spectral functions, corresponding to a given Feynman diagram, are non-zero. Finally, it is demonstrated that the unproven assumption of the double dispersion relations was not really a necessary step: the perturbation theory terms could be generated from unitarity together with rigorously proven analyticity properties only.

The import of this paper is to confirm the contention that dispersion relations together with unitarity contain as much information as Lagrangian perturbation theory.

J. C. Taylor (Rochester, N.Y.)

577:

O'Raifeartaigh, L.; Średniawa, B.; Terreaux, Ch. The proton-neutron mass-difference according to meson-theory. Nuovo Cimento (10) **14** (1959), 376–396. (Italian summary)

The neutron-proton mass difference is calculated to order e^2f^2 in a cut-off ps-ps and ps-pv meson theory, keeping only terms or order $1/M$ (M = nucleon mass). The gauge-invariant cut-off is such as to assign a further factor $m^2K^2(m^2K^2+(p\cdot q)^2-p^2q^2)^{-1}$ to any vertex at which a boson of momentum $p-q$ is emitted or absorbed where m is the mass of the particle emitting or absorbing the boson, p and q the momenta of the particle before and after the act of emission or absorption and K is the cut-off momentum (assumed of order $1/M$). A strongly cut-off dependent mass difference of the correct sign but too small by at least an order of magnitude is obtained for both the pseudoscalar and the pseudovector coupling. The pv coupling results are somewhat larger but still too small to

overcompensate the purely electromagnetic self-energy of the proton. *S. Schweber* (Waltham, Mass.)

578:

Lévy, M. On the validity of the exponential law for the decay of an unstable particle. Nuovo Cimento (10) **14** (1959), 612–624. (Italian summary)

Author's summary: "The departure from the exponential decay law of an unstable particle is discussed, with the help of the Lee model, starting from an initial state which is a general superposition of eigenstates of the Hamiltonian. It is shown that the asymptotic behaviour and the magnitude of the additional terms are strongly affected by the production mechanism; although they can never vanish exactly, these terms can be made arbitrarily small by an appropriate preparation of the initial state. A possible generalization of this result is discussed."

579:

Šapiro, I. S. Symmetrieeigenschaften in der Theorie der Elementarteilchen und Kernprozesse. Fortschr. Physik **7** (1959), 243–289.

This is a translation of a review article first published in Russian [I. S. Šapiro, Uspehi Fiz. Nauk **53** (1954), 7–68; MR **16**, 321]. Contained therein are discussions of the concepts of space inversion (preparity non-conservation), time reversal, charge conjugation, and isotopic spin. The use of the concepts is in most cases illustrated by the derivation of selection rules for various transition processes. Fuller, more general, and in some instances more transparent discussions of these topics can be found in the more recent literature [e.g., Roman, *Theory of elementary particles*, North-Holland Publ. Co., Amsterdam; Interscience Pub. Inc., New York, 1960].

A. Klein (Philadelphia, Pa.)

580:

Lee, T. D.; Yang, C. N. Many-body problem in quantum statistical mechanics. III. Zero-temperature limit for dilute hard spheres. Phys. Rev. (2) **117** (1960), 12–21.

The theory of earlier papers in this series [Phys. Rev. (2) **113** (1959), 1165–1177; **116** (1959), 25–31; MR **21** #1725; #7039] is extended and the thermodynamic properties of a dilute hard-sphere gas are evaluated at the absolute zero, both for fermions where one can immediately evaluate the pressure, and hence all other thermodynamic properties, and for bosons where the situation is more complicated because of the Einstein condensation. To circumvent this difficulty it is pointed out that the thermodynamic properties of a Boltzmann gas at absolute zero are the same as those of a Bose gas, and those of a Boltzmann gas are easily evaluated.

D. ter Haar (Oxford)

581:

Lee, T. D.; Yang, C. N. Many-body problem in quantum statistical mechanics. IV. Formulation in terms of average occupation number in momentum space. Phys. Rev. (2) **117** (1960), 22–36.

The fourth paper in this series [see preceding review] deals with the grand partition function of a system of interacting particles. The method outlined in the first part of this series is used. The thermodynamic properties are expressed in terms of the statistical averages of the occupation numbers in momentum space, and in terms of a simple variational problem. The formalism is applied to systems of hard spheres, both bosons and fermions.

D. ter Haar (Oxford)

582:

Hiroike, Kazuo. A formal theory of collective behavior. Progr. Theoret. Phys. **23** (1960), 41–60.

Bogolyubov and Zubarev's method of auxiliary variables [Ž. Èksper. Teoret. Fiz. **28** (1955), 129–139; MR **17**, 113] is generalized so as to be applicable to systems with strong interactions between the particles and to fermion systems. This is done by splitting up the potential energy into two parts, one containing all strong interactions. The theory is developed formally, but possible applications to a hard sphere boson gas, a fermion gas with weak interactions, and a classical fluid are briefly discussed.

D. ter Haar (Oxford)

583:

Alekseev, A. I. Nonrelativistic solution of the Bethe-Salpeter equation. Ž. Èksper. Teoret. Fiz. **36** (1959), 1435–1437 (Russian); translated as Soviet Physics. JETP **9**, 1020–1021.

The solution of the Bethe-Salpeter equation for two identical Dirac particles is written down through terms of order v/c, corresponding to the usual large component-small component decomposition of the one-particle Dirac equation. No applications are made.

H. W. Lewis (Madison, Wis.)

584:

Fujita, S. Pair distribution function and two-body propagator. Phys. Rev. (2) **115** (1959), 1335–1341.

Author's summary: "Starting from the ordinary definition of the pair distribution function, the following theorem is proved without the help of the cluster integral expansion and "toron" diagrams: the pair distribution function in the grand canonical ensemble can be expressed in terms of the two-body propagator corresponding to the scattering in position-reciprocal temperature space. A simple expression for the internal energy of a nonideal gas with pair interactions is obtained in terms of the pair distribution function." *A. Klein* (Philadelphia, Pa.)

585:

Kröner, E. Zur Behandlung des quantenmechanischen Vielteilchenproblems mit Hilfe von Zweiteilchenfunktionen. Z. Naturf. **15a** (1960), 260–265.

Zusammenfassung des Autors: "Nach Bopp ist jedem quantenmechanischen Problem von N gleichartigen symmetrischen Teilchen, die aufeinander nur mit Zweiteilchenkräften wirken, in Strenge ein Zweiteilchenproblem zugeordnet. Je $N/2$ Lösungen φ_n dieses Problems werden zu Produkten zusammengefaßt, aux denen durch Antisymmetrisierung Konfigurationen gewonnen werden. Die zugehörige Reihenentwicklung sollte beträchtlich besser konvergieren als alle auf Einteilchenfunktionen basierenden Entwicklungen. Auftretende Schwierigkeiten bei der Berechnung der zu den Konfigurationen gehörigen Energie-Matrix-elemente und Überlappintegrale können weitgehend überwunden werden."

586:

Munschy, Gilbert; Pluvinage, Philippe; Proriol, Joseph. Fonctions de base de symétrie définie pour les fonctions d'onde d'un système de trois particules identiques. J. Phys. Radium **21** (1960), 85–93. (English summary)

Authors' summary: "As a first step in the precise study of the quantum radial problem for a system of three identical particles, we give a method of building orthogonal sets of eigenfunctions with definite character of symmetry. Rather than the three linear distances of the particles, three variables more closely related to a representation of the symmetric group S_3 are chosen. The first is not affected by permutations. The two others are transformed in a simple way, and it is possible to form with them polynomials of definite symmetry. These polynomials are determined as eigenfunctions of two commutable operators. Explicit expressions are given up to the fifth degree, what seems quite sufficient for practical purposes."

587:

Wess, J. On scale transformations. Nuovo Cimento (10) **14** (1959), 527–531. (Italian summary)

Author's summary: "Invariance against scale transformation was used by W. Heisenberg to define a lepton number. Some properties of this transformation are more easily investigated with scalar fields, the mass dependence of which is known. For the scalar fields and the harmonic-oscillator, some properties of the infinitesimal transformation are listed here."

588:

Martin, Paul C.; Schwinger, Julian. Theory of many-particle systems. I. Phys. Rev. (2) **115** (1959), 1342–1373.

This paper describes the foundations for a treatment of the problems of statistical thermodynamics and transport phenomena by the methods of field quantization in the Heisenberg representation. After a concise review of some of the foundations of statistical mechanics, the fundamental elements of the method, the Green's functions or correlation functions are introduced in forms appropriate to a discussion of either the microcanonical or grand canonical ensembles. It is remarked that most observables of interest are readily expressible in terms of either the one- or two-particle Green's functions. Spectral representations of the Källen-Lehman type are obtained for the one-particle Green's function. As an example of the applications to non-equilibrium processes, a restricted version of the two-particle Green's function appropriate to the discussion of current correlations is studied and various well-known results such as the fluctuation-dissipation theorem, a sum rule for conductivity, and certain dispersion relations are rederived. The core of the method involves the possibility of obtaining non-perturbative solutions to the infinite set of coupled equations satisfied by the Green's functions. It is first demonstrated that boundary conditions can be prescribed quite independently of any adiabatic decoupling. A method of solution is then suggested which amounts to assuming an upper limit on the number of excitations whose motion can be simultaneously correlated with one another. In first approximation one obtains a generalized Hartree-Fock equation. An approximate form of the two-particle correlation approximation is also treated in considerable detail, and this permits among other points, a discussion of finite lifetime effects for elementary excitations. The paper concludes with an elaborate discussion of formal solution methods and includes alternative "derivations" of the method of successive correlations and of methods (the generalized dielectric constant approach) for the treatment of collective excitations. A clear presentation and physical justification of the exact manner in which the approximations are to be chosen is lacking, but perhaps will be clarified by the papers on applications which are promised. Nevertheless in its broad outlines the paper promises a program which is more generally conceived than that of various other recent papers (referenced in the present paper) that have introduced analogous methods into statistical physics.

A. Klein (Philadelphia, Pa.)

589:

Mühlschlegel, Bernhard. Die thermodynamischen Funktionen des Supraleiters. Z. Physik **155** (1959), 313–327.

In this paper the author shows that the thermodynamics of the Bardeen, Cooper, Schrieffer theory of superconductivity can be obtained in terms of a single function $\alpha(x)$ and its derivatives. He shows how, from this function, the free energy, specific heat and other thermodynamic quantities can be obtained. Newly calculated tables of thermodynamic functions are given.

L. N. Cooper (Providence, R.I.)

590:

Gupta, K. K.; Mathur, V. S. Electromagnetic properties of superconductors. Phys. Rev. (2) **115** (1959), 75–78.

Using Wentzel's explicitly gauge invariant derivation of the expression for the current density in the BCS model of superconductivity, and neglecting all temperature and Coulomb effects, the authors obtain a nonlocal relationship between the current and the vector potential similar to that first proposed by Pippard. Their results differ from that of other gauge invariant calculations by constant factors. The differences between the various methods are due to different partial summations of infinite series.

L. N. Cooper (Providence, R.I.)

591:

Swihart, James C. Isotope effect in the Bardeen-Cooper-Schrieffer and Bogoliubov theories of superconductivity. Phys. Rev. (2) **116** (1959), 45–52.

In this paper it is shown that recent theories of superconductivity predict an isotope effect which is the same for all superconductors for a certain class of electron-electron interactions. The usually chosen electron-electron interaction due to the exchange of phonons, as derived for example by Bardeen and Pines, falls into the proper class. The Coulomb interaction, however, does not. The author shows that the theory predicts that H_0, T_c, and the energy gap at $T=0$ are proportional to $M^{-1/2}$. The inclusion of the Coulomb interaction destroys invariance of the equations and introduces deviations from the $-1/2$ in the exponent. The magnitude of the deviations depends on the particular superconductor considered.

L. N. Cooper (Providence, R.I.)

592:

Moskalenko, V. A. On the thermodynamics of superconductivity. Soviet Physics. Dokl. **123** (3) (1958), 1171–1175 (433–436 Dokl. Akad. Nauk SSSR).

The thermodynamics of the Bardeen, Cooper, Schrieffer theory of superconductivity are reproduced in a somewhat more general manner by proceeding directly from the Fröhlich-Hamiltonian. *L. N. Cooper* (Providence, R.I.)

593:

Wada, Yasushi; Fukuda, Nobuyuki. On the theory of superconductivity. Progr. Theoret. Phys. **22** (1959), 775–806.

Authors' summary: "Bardeen, Cooper and Schrieffer's theory of superconductivity is discussed by developing an exact treatment of strong coupling approximation. With respect to the thermal properties, their results are verified: The energy of the ground state and the magnitude of the energy gap are found to be identical with the exact answers in the strong coupling limit. However, the argument for the magnetic properties such as the Meissner effect seems to leave some questions. We have discussed the Meissner effect by means of the gauge invariant method presented in the previous paper [Prog. Theoret. Phys. **21** (1959), 343–353; MR **21** #3245]. In order to maintain the invariance exactly the case of strong coupling between electrons is investigated. Then the "interaction current" plays an essential role and is shown to diverge with the size of the system. In view of the qualitative resemblance of the BCS variational solution to that of the strong coupling approximation, it may be inferred that the above conclusion seems to hold also in the weak coupling case. This will suggest the importance of the interaction current and the requirement of the exact gauge invariance contrary to Anderson's arguments."

K. Gottfried (Cambridge, Mass.)

RELATIVITY

594:

Strauss, Martin. Grundlagen der Kinematik. I. Die Lösungen des kinematischen Transformationsproblems. Wiss. Z. Humboldt-Univ. Berlin. Math.-Nat. Reihe **7** (1957/58), 609–616. (English, Russian and French summaries)

Kritische Betrachtungen über die gruppentheoretische Begründung der speziellen Relativitätstheorie. Verf. gibt eine Darstellung, wobei die Geschwindigkeit als Grundbegriff auftritt, während die üblicherweise zur Definition der Geschwindigkeit verwendete Gleichung zur Definition der Systemzeitmetrik verwendet wird. Er geht von gewissen Postulaten aus und findet daraus als Lösung für die Komposition kollinearer Geschwindigkeiten die Formel der Lorentz-Transformation und diejenige mit c^2 statt $-c^2$; von dieser Alternativlösung wird eine Deutung gegeben. {Die Resultate scheinen dem Ref. nicht einwandfrei; so wird behauptet, dass die Funktionalgleichung

$$g(x_1+x_2)/(1-x_1x_2)=g(x_1)+g(x_2)$$

nur die Lösung $g=0$ hat, während doch offensichtlich $g(x)=\operatorname{arctg} x$ eine Lösung ist.} *O. Bottema* (Delft)

595:

Törnebohm, Håkan. The space-time theory within the special theory of relativity. An essay in reconstructive analysis. Methodos **10** (1958), 243–266.

The author attempts to give a logical reconstruction of the foundations of special relativity in a manner which has some affinity with that due to A. A. Robb, except that he uses a definition for the simultaneity of distant events analogous to Einstein's. His proof of the Lorentz formulae depends, however, on the Lorentz invariance of $(t^2-x^2/c^2)^{1/2}$, in the usual notation, and his derivation of this invariance appears to be faulty. For he only proves that the equation $(t^2-x^2/c^2)^{1/2}=0$ is invariant and not the expression $(t^2-x^2/c^2)^{1/2}$. *G. J. Whitrow* (London)

596:

Kronsbein, John; Farber, Erich A. Time retardation in static and stationary spherical and elliptic spaces. Phys. Rev. (2) **115** (1959), 763–764.

The authors show that it is possible to resolve the clock paradox of special relativity within a context which involves no reference to accelerations by considering motion in a closed static space. (This was first shown by E. A. Milne in a joint paper with the reviewer [Phil. Mag. (7) **40** (1949), 1244–1249; MR **11**, 409; cf. p. 1248].)

G. J. Whitrow (London)

597:

Builder, Geoffrey. Resolution of the clock paradox. Amer. J. Phys. **27** (1959), 656–658.

The author shows that there is no logical antinomy associated with the so-called clock paradox of special relativity because the second clock is not associated with the same inertial frame of reference throughout. (This point has, of course, been made by others.)

G. J. Whitrow (London)

598:

Crampin, Joan; McCrea, W. H.; McNally, D. A class of transformations in special relativity. Proc. Roy. Soc. London. Ser. A **252** (1959), 156–176.

M. Born and W. Biem [Nederl. Akad. Wetensch. Proc. Ser. B **61** (1958), 110–120; MR **20** #736] introduced a class of transformations from the coordinates used by an observer A in an inertial frame to those used by an accelerated observer B. The authors study the properties of this transformation in greater detail and with more emphasis on mathematical existence theorems. Particular attention is paid to the case in which B's world-line in A's coordinate system is part of a hyperbola meeting A's world-line twice. The example usually quoted in discussions of the clock paradox is treated as a limiting case of this. The paper concludes with some pertinent comments on Tolman's well-known appeal to general relativity in order to resolve this paradox. The authors show that this appeal is unnecessary.

G. J. Whitrow (London)

599:

Costa de Beauregard, Olivier. Sur l'utilisation du tenseur d'impulsion-énergie électromagnétique de Louis de Broglie en électromagnétisme classique. C. R. Acad. Sci. Paris **249** (1959), 2285–2286.

The author compares different forms for the stress tensor of the electromagnetic field.

D. W. Sciama (London)

600:

Newman, E.; Goldberg, J. N. Measurement of distance in general relativity. Phys. Rev. (2) **114** (1959), 1391–1395.

The proposed distance is proportional to the separation, in an observer's rest frame, between two neighbouring null geodesics representing light rays emitted simultaneously by a remote object. The authors remark that in cosmological models their definition agrees with Whittaker's luminosity distance [W. O. Kermack, W. H. McCrea, and E. T. Whittaker, Proc. Roy. Soc. Edinburgh **52** (1933), 31–47] and assert that it is physically identical. {It appears to the reviewer that in space-times without symmetry the Whittaker distance will be an average, over orientations perpendicular to the line of sight, of the Newman-Goldberg distance, which will be orientation-dependent.} *F. A. E. Pirani* (London)

601:

Lowry, Edward S. Geometrical representation of the Maxwell field in Minkowski space. Phys. Rev. (2) **117** (1960), 616–618.

An electromagnetic field may be considered to be the superposed fields of charges described by tensors of the type

$$f_{ij} = \frac{e[u_i + (u_i \times a_k)c_k] \times c_j}{4\pi(u_m c_m)^3},$$

where u_i, a_i are the velocity and acceleration 4-vectors of a charge e, and c_i is the null-vector from the charge to the field event [cf. C. Møller, *The theory of relativity*, Clarendon, Oxford, 1952; MR **14**, 212; p. 150]. The author shows the equivalence of this representation of the field to a geometrical representation in Minkowski space, by associating the tensor f_{ij} with the orientation and density of a family of two dimensional surfaces radially distributed about the world line of the charge. He suggests that, if possible, it may be advantageous to describe other fundamental fields as structures of subsets of points in the space, rather than as tensor, or spinor functions.

C. Gilbert (Newcastle-upon-Tyne)

602:

Papapetrou, Achilles; Treder, Hansjürgen. Das Sprungproblem erster Ordnung in der allgemeinen Relativitätstheorie. Math. Nachr. **20** (1959), 53–66.

The discontinuities in the gravitational field were discussed in great detail by K. Stellmacher [Math. Ann. **115** (1938), 740–783] from the point of view of the theory of the characteristics of Einstein's field equations, the discontinuities of both the first and second derivatives of the metric tensor $g_{\mu\nu}$ being considered. Recently H. Treder [Ann. Physik (7) **2** (1958), 225–235; MR **21** #4808] devised an elegant simplified method by means of which the same problem may be treated. This method is applied in the present paper to a discussion of the discontinuities of the first derivatives of $g_{\mu\nu}$, for which Stellmacher did not obtain unique results (as contrasted with his results for the second derivatives). It is shown that this difficulty resulted from the assumption of the existence of sources of the gravitational field distributed over the surfaces of discontinuity. When this assumption is excluded the discontinuities of the first order are, in fact, uniquely determined and possess the same properties as those of the second order, while discontinuities of the first order which cannot be removed by mere transformations can occur only on the characteristic surfaces of the field equations. However, discontinuities of the first derivatives of the $g_{\mu\nu}$ lead to discontinuities of the connection coefficients and therefore affect the geodesics, but it is found that the differential equations of the bicharacteristics are the same on both sides of the surface of discontinuity. The method is extended to the case where an electromagnetic field is present and again it is found that the discontinuities of the first derivatives of $g_{\mu\nu}$ and of the 4-potential are uniquely determined and possess the same properties as the discontinuities of the second derivatives which had been determined by Stellmacher. *H. Rund* (Durban)

603:

Schmutzer, Ernst. Zur Theorie der Spinoren im Riemannschen Raum. Z. Naturf. **15a** (1960), 355–362.

Zusammenfassung des Autors: "Die Theorie der Spinoren im Riemannschen Raum von Infeld und van der Waerden wird mit Hilfe der bereits in der projektiven Relativitätstheorie bewährten Methode der Basisvektoren weiterentwickelt. Die spinorielle Geometrie läßt sich in eleganter Weise bis zur Ableitung von grundlegenden Identitäten analog den Bianchi-Identitäten der tensoriellen Geometrie aufbauen. Es ergeben sich mehrere wichtige Beziehungen für den gekrümmten Spinorraum. Während in der üblichen Theorie die Wurzel des elektromagnetischen Feldes im Nichtverschwinden der kovarianten Ableitung des metrischen Spinors gesehen wird, da sich keine andere Interpretationsmöglichkeit zu bieten scheint, kann gezeigt werden, daß auch die wesentlich vereinfachte Geometrie mit verschwindender kovarianter Ableitung des metrischen Spinors zwangsläufig auf einen antisymmetrischen Tensor führt, der dem elektromagnetischen Feld zugeordnet wird."

604:

Misra, S. P. Quantum mechanics in curved space-time. Progr. Theoret. Phys. **23** (1960), 1–16.

The author discusses here the necessary modification of the first order equations (Dirac, Kemmer, Bhabha equations) of quantum mechanics for a weak gravitational field. Corrections are obtained in a coordinate system which is locally Galilean, and illustrated for particular systems. The connection with affine spinor theories is discussed. *H. S. Green* (Adelaide)

605:

Peres, A.; Rosen, N. On Cauchy's problem in general relativity. Nuovo Cimento (10) **13** (1959), 430–438. (Italian summary)

A rediscussion of the title problem for the vacuum [for the standard discussion see A. Lichnerowicz, *Théories relativistes de la gravitation et de l'électromagnétisme*, Masson, Paris, 1955; MR **17**, 199]. The authors' attempt to free their arguments of coordinate dependence by considering tetrad components of the Riemann tensor is

inadequate because they do not fix the reference tetrad in an invariant way. *F. A. E. Pirani* (London)

606:

Fourès-Bruhat, Yvonne. Mécanique des fluides relativistes. Cahiers de Phys. **13** (1959), 463–468.

In an earlier paper [Bull. Soc. Math. France **86** (1958), 155–175; MR **21** #4036] the author has discussed the existence of well posed initial value problems in general relativity. In this note the author gives a survey of her work. A new result is also quoted according to which the equation system describing a gravitational field, an electromagnetic field and a charged fluid in motion does not lead to a well posed initial value problem if the fluid has a finite electric conductivity. {This is astonishing since in the case of weak gravitational fields no such anomalies arise.} *N. L. Balazs* (Princeton, N.J.)

607:

Klotz, A. H. Gravitation and electromagnetism. Nuovo Cimento (10) **14** (1959), 135–141. (Italian summary)

A unified field theory is suggested with "metric"

$$ds^2 = a_{\mu\nu}dx^\mu dx^\nu + \phi_{\mu\nu}d^{(3)}s^{\mu\nu},$$

where the $s^{\mu\nu}$ are skew symmetric tensor elements whose meaning is somewhat obscure. The gravitational equations of empty space, together with one set of Maxwell's equations, are derived from an affine relation

$$\delta A^\alpha = -\Gamma_{\mu\nu}{}^\alpha A^\mu \delta x^\nu - F_\mu{}^\alpha A^\mu \delta s.$$

Variational principles are discussed and reference is made to an empirical test, but with no indication of the numbers involved. *C. W. Kilmister* (London)

608:

Urban, A. On space of an unified field theory of gravitation and electromagnetism. Tensor (N.S.) **9** (1959), 205–208.

Let

$$\Gamma^\nu_{\lambda\mu} = \{^\nu_{\lambda\mu}\} + \phi_\mu \delta^\nu_\lambda \tag{1}$$

be the connection used by Husain [Tensor (N.S.) **6** (1956), 132–135; MR **18**, 543]. Then the third identity for the curvature tensor $R_{\omega\mu\lambda}{}^\nu$ yields

$$V_{\omega\mu} \overset{\text{def}}{=} R_{\omega\mu\lambda\nu}g^{\lambda\nu} = 2n\partial_{[\omega}\phi_{\mu]}. \tag{2}$$

Moreover, because the connection (1) is obviously semi-symmetric

$$S_{\mu\lambda}{}^\nu = S_{[\mu}\delta^\nu_{\lambda]}$$

one obtains easily $S_\lambda = \phi_\lambda$ and

$$R_{[\mu\ \ \omega]}{}^\alpha{}_\nu \overset{\text{def}}{=} R_{\alpha[\mu\nu]}{}^\alpha = -2\partial_{[\mu}\phi_{\lambda]}.$$

V. Hlavatý (Bloomington, Ind.)

609:

Mishra, R. S. n-dimensional considerations of unified theory of relativity. Recurrence relations. Tensor (N.S.) **9** (1959), 217–225.

The reviewer solved the Einstein equations for the unified connection by using the inherent non-holonomic frame induced in the space-time by the fundamental tensor $g_{\lambda\mu} \neq g_{\mu\lambda}$ [Hlavatý, *Geometry of Einstein's unified field theory*, P. Noordhoff, Groningen, 1957; MR **20** #5067]. Wrede [Tensor (N.S.) **8** (1958), 95–122; MR **20** #5073] generalized this method for an arbitrary n. The author of the present paper uses some of Wrede's results to deal with the recurrent relations for the tensors ${}^{(p)}k_\lambda{}^\nu$. Here $k_{\lambda\mu}$ is the skew symmetric part of $g_{\lambda\mu}$, $k_\lambda{}^\nu = k_{\lambda\mu}h^{\mu\nu}$, where $h^{\lambda\mu}$ is universe to the symmetric part of $g_{\lambda\mu}$ and ${}^{(p)}k_\lambda{}^\nu$ is the product of p factors:

$${}^{(p)}k_\lambda{}^\nu \overset{\text{def}}{=} k_\lambda{}^{\alpha_1}k_{\alpha_1}{}^{\alpha_2} \cdots k_{\alpha_{p-1}}{}^\nu.$$

He finds the lowest order recurrence relation for the first class as well as for the second class of category j. Moreover he finds necessary and sufficient conditions for the second class of category j as well as for the third class. Example: Necessary and sufficient conditions for the third class are ${}^{(r)}k_\mu{}^\nu = 0$ $(r = 3, 4, \cdots)$. The restrictive condition imposed on $g_{\lambda\mu}$ is that the secular equation belonging to $k_\lambda{}^\nu$ does not have multiple non-zero roots.

V. Hlavatý (Bloomington, Ind.)

610:

Dautcourt, Georg. Sur la solution de l'équation d'Einstein $g_{\mu\ \ ;\rho}^{\ \nu\ \ +\ -} = 0$. C. R. Acad. Sci. Paris **249** (1959), 2159–2161.

(The numbers in parenthesis refer to the pages of the book by Hlavatý, *Geometry of Einstein's unified field theory* [Noordhoff, Groningen, 1957; MR **20** #5067].)

(1) Let $g_{\lambda\mu} \neq g_{\mu\lambda}$ be the fundamental tensor of Einstein's Unified Theory, $h_{\lambda\mu}$ $[k_{\lambda\mu}]$ its symmetric [skew symmetric] part, ${}^*g^{\lambda\mu}$, $h^{\lambda\mu}$ the inverse of $g_{\lambda\mu}$, $h_{\lambda\mu}$, respectively.

$$g_{\lambda\mu}{}^*g^{\lambda\nu} = h_{\lambda\mu}h^{\lambda\nu} = \delta_\mu{}^\nu.$$

Then the tensors

$$k_\lambda{}^\nu \overset{\text{def}}{=} k_{\lambda\mu}h^{\mu\nu}, \quad \overset{-1}{T}{}_\lambda{}^\nu = g_{\lambda\mu}{}^*g^{\mu\nu}$$

have the same eigenvectors and the corresponding eigenvalues λ and ρ of k and $\overset{-1}{T}$ are related by $\rho = (1+\lambda)/(1-\lambda)$ (12, 13). If in particular $g_{\lambda\mu}$ is not of the third class there are four eigenvectors $\overset{j}{a}{}^\nu$ which are null vectors. Denoting by $\overset{j}{a}_\lambda$ the dual system, one obtains for the non-holonomic coordinates of $k_\lambda{}^\nu$ and $\overset{-1}{T}{}_\lambda{}^\nu$ in this non-holonomic frame (a) $k_x{}^j = \underset{x}{\lambda}\delta_x{}^j$, (b) $\overset{-1}{T}{}_x{}^j = \rho_x\delta_x{}^j$, (c) $\rho_x = (1+\underset{x}{\lambda})/(1-\underset{x}{\lambda})$ (no sum).

(2) The reviewer solved the Einstein equations for the unified connection using (a) (78, 83). The author finds the solution using (b). He claims that his method does not use the decomposition of $g_{\lambda\mu}$. However the equations (a)–(c) show that using (b) amounts substantially to the use of (a), (c).

(3) The third class requires another approach (86) because the equations (a)–(c) do not hold. Hence the author's method can be applied only to the first two classes.

{There is a misprint in the author's equation (3).}

V. Hlavatý (Bloomington, Ind.)

611:

Bkouche, Rudolphe. Sur la métrique asymétrique. C. R. Acad. Sci. Paris **249** (1959), 2282–2284.

Résumé de l'auteur: "On établit, à partir d'une étude générale, des propriétés de la métrique asymétrique, indépendantes de la dimension. On retrouve une relation établie dans le cas particulier de l'espace R^4 par une méthode qui fait intervenir explicitement la dimension 4."

V. Hlavatý (Bloomington, Ind.)

612:

Yamamoto, Hiroshi. Approximate solution of the relativistic two-body equation and its application to the nuclear forces. Progr. Theoret. Phys. **22** (1959), 73–88.

A model of the ladder approximation Bethe-Salpeter equation is studied, in which the meson exchange potential is replaced by a separable potential. The exact solutions of this model equation are employed to investigate the validity of non-relativistic approximations.

J. C. Taylor (Rochester, N.Y.)

ASTRONOMY

See also 329, 347.

613:

Limber, D. Nelson. Effects of intracluster gas and dust upon the virial theorem. Astrophys. J. **130** (1959), 414–428.

The usual form of the virial theorem for a steady, self-gravitating system involves the kinetic and gravitational energies of all the matter present; it should not be applied to a cluster of galaxies (or stars) which contains a significant fraction of its mass as intracluster gas or dust, unless their contributions to both energies are included. In this paper a new steady-state virial theorem is deduced: the mean-square velocity of the galaxies, weighted according to their masses, is given by

$$\langle V_G^2 \rangle = C_{GG}\frac{GM_G}{R}\left[1+\left(\frac{C_{Gg}+D_{Gg}}{C_{GG}}\right)\frac{M_g}{M_G}\right].$$

Here M_G and M_g are respectively the total masses of all the galaxies and all the intracluster gas, G the gravitational constant, and R a suitably defined cluster radius. C_{GG} depends only on the spatial distribution of the galaxies, as in the absence of gas; C_{Gg} and D_{Gg} depend on the distribution of both gas and galaxies, D_{Gg} vanishing if the distributions are geometrically similar.

For practical use of the theorem, it is assumed that the two densities can be expanded as sums of multiples of the six polytropic distributions of indices 0, 1, 2, 3, 4, 5, with a common radius R. Basic quantities are tabulated for the thirty-six possible pairs of polytropic distributions with these indices.

If M_g is assumed to vanish, the theorem is usually applied to compute M_G from observations of $\langle V_G^2 \rangle$, C_{GG} and R. If $M_g \neq 0$, and the gas is neutral hydrogen, optically thin at 21 cm., then in principle M_g and $(C_{Gg}+D_{Gg})$ can be obtained and M_G estimated. (Presumably 21 cm. observations would supply information on the turbulent energy as well as the density of the gas, and so would allow application of the usual virial theorem.)

L. Mestel (Cambridge, England)

614:

Pinto, R.; Simon, R. Sur les oscillations radiales et la stabilité d'un plasma cylindrique. Acad. Roy. Belg. Bull. Cl. Sci. (5) **45** (1959), 595–610. (English summary)

In its equilibrium state, a homogeneous infinite cylinder of plasma is pervaded by a uniform longitudinal magnetic field $\mathbf{B}_p$. The vacuum surrounding it has a uniform field $\mathbf{B}_v$ parallel to $\mathbf{B}_p$, and a curl-free toroidal field, of strength varying inversely with the radial distance from the cylinder axis. Sheet currents flow on the plasma surface, maintaining the discontinuities in the (tangential) components of the field. The problem differs from the usual "pinch-effect" [M. Kruskal and M. Schwarzschild, Proc. Roy. Soc. London. Ser. A **223** (1954), 348–360; MR **15**, 914; M. Kruskal and J. L. Tuck, ibid. **245** (1958), 222–237; MR **20** #626] in that account is taken of the self-gravitation of the plasma.

The stability of the system against small radial perturbations $\propto e^{i\omega t}$ is discussed, assuming infinite electrical conductivity, a scalar pressure, and adiabatic pressure changes. The possible values of ω satisfy a transcendental equation involving hypergeometric and Bessel functions. The variation with the parameters of the system of the particular value of ω near $\omega=0$ is shown to divide the physically allowed parameter values into two regions, one yielding stability, the other instability for this particular mode. Because of the extreme difficulty of a general discussion of the behaviour of the other modes, the authors advance the hypothesis that these two regions correspond respectively to stability and instability for all the modes, so deriving a necessary and sufficient condition for stability.

The results of the paper depend on the equilibrium state being of uniform density, in spite of its self-gravitation and the resulting pressure gradient. This requires that there be a temperature gradient, maintained against equalising tendencies such as radiative cooling and thermal conduction. In an actual astrophysical context it is more likely that the temperature will be nearly uniform; the equilibrium state would then possess a density gradient, and the stability problem would need to be tackled again.

L. Mestel (Cambridge, England)

GEOPHYSICS

See also 397, 399, 423.

615:

Henderson, Roland G. A comprehensive system of automatic computation in magnetic and gravity interpretation. Geophysics **25** (1960), 569–585.

616:

Scheube, Hans-Georg. Die Lösungen der Dirichletschen und Neumannschen Randwertaufgaben als Hilfsmittel zur Behandlung von Problemen des Erdmagnetismus. Abh. Akad. Wiss. Göttingen. Math.-Phys. Kl. Suppl. no. 4 (1958), 1–32.

Formulae are given for the approximate decomposition of a local anomaly in the magnetic field at the earth's surface into components where origin is either external or internal. Examples are given.

A. Herzenberg (Manchester)

617:
Siebert, Manfred. Die Zerlegung eines lokalen erdmagnetischen Feldes in äusseren und inneren Anteil mit Hilfe des zweidimensionalen Fourier-Theorems. Abh. Akad. Wiss. Göttingen. Math.-Phys. Kl. Suppl. no. 4 (1958), 33–38.

A general potential field satisfying Laplace's equation whose derivatives are given on an infinite plane is decomposed into components whose sources are on one or the other side by means of integrals over the plane.

A. Herzenberg (Manchester)

618:
Rikitake, Tsuneji. Forced oscillations of the earth's dynamo. Bull. Earthquake Res. Inst. Tokyo **37** (1959), 245–264. (Japanese summary)

619:
Rikitake, Tsuneji. Thermo-magneto-hydrodynamic oscillations in the earth's core. Bull. Earthquake Res. Inst. Tokyo **37** (1959), 405–422. (Japanese summary)

620:
Hallert, B.; Ottoson, L. General differential formulae of the complete projective relations between planes. Kungl. Tekn. Högsk. Handl. Stockholm. No. 140, 15 pp. (1959).

In this paper the authors, starting from the well known formulae between the coordinates of corresponding points of two planes in perspective relation, derive the differential formulae for arbitrary mutual positions of the two planes and for different arrangements of the axes of rotations and of the sequence of these rotations, with regard to aerial and terrestrial photogrammetry.—They point out the many important applications of the differential formulae given in this paper to various problems, in particular in photogrammetry.—For instance, considering the case of approximately vertical photographs and that of convergent photographs, particular differential formulae are given, which are of fundamental importance for the correct treatment of the theory of errors in aerial photogrammetry. *M. Piazzolla-Beloch* (Ferrara)

621:
Liu, Hui-lin. Adjustment of triangulation net by directions: estimation of precision. Acta Geodet. Cartograph. Sinica **3** (1959), 40–54. (Chinese)

622:
Li, Ching-hai. The error-ellipse of forward intersection and the probability of longitudinal and transverse errors. Acta Geodet. Cartograph. Sinica **3** (1959), 116–125. (Chinese. English summary)

623:
Ledersteger, K. Zu den Beweisen des Theorems von Lichtenstein und der Unmöglichkeit ähnlicher Schichtung. Schweiz. Z. Vermessg. Kulturtech. Photogr. **58** (1960), 1–10.

OPERATIONS RESEARCH, ECONOMETRICS, GAMES

See also A202, 305, 316, 335.

624:
Azorín Poch, Francisco. Statistical considerations on the Cobb-Douglas function. Estadíst. Española no. **4**, 19–29 (1959). (Spanish. English summary)

The author considers the Cobb-Douglas function of the form

$$y(x) = \alpha x^{\beta} \tag{1}$$

relating an "output" y to an "input" x, with special reference to estimating the "marginal productivity"

$$\frac{dy}{dx} = \beta y(x)/x \tag{2}$$

from associated input and output data, x_i, y_i, $i = 1, \cdots, n$. The customary procedure for estimating $\beta y(x)/x$ for a given fixed input x is to assume that the logarithmic form of (1),

$$v(u) = \ln y(x) = \ln \alpha + \beta \ln x = \alpha' + \beta u, \tag{3}$$

has independent, equal-variance residuals. The estimate of (2) arising from this assumption is

$$b\mathcal{Y}(x) = b \exp(a' + bu), \tag{4}$$

where a' and b are the familiar regression estimates of α' and β. Carter and Hartley [Econometrica **26** (1958), 306–313; **20** #3743] derived an approximate variance for $b\mathcal{Y}(x)$. The author derives a formula for the bias of $b\mathcal{Y}(x)$ (not a bias for the Carter-Hartley variance estimate as he inadvertently states). He says that this bias is not negligible compared with the standard deviation of $\mathcal{Y}(x)$, but actually it is negligible for large n.

L. J. Savage (Ann Arbor, Mich.)

625:
Mandelbrot, Benoît. Variables et processus stochastiques de Pareto-Lévy, et la répartition des revenus. C. R. Acad. Sci. Paris **249** (1959), 2153–2155.

This is a brief note on certain ideas in connection with the much-discussed problem of how to find an analytical expression for the distribution of incomes.

T. Haavelmo (Oslo)

626:
Operations Research Center, M.I.T. ★Notes on operations research 1959. The Technology Press, Cambridge, Mass., 1959. viii + 256 pp.

The eleven chapters of these notes provided basic material for lectures on operations research given to persons from European NATO countries. They cover probability, search, Markov processes, queuing, control processes, organization of operations research groups, sequential decision processes, reliability and maintenance, information theory, production scheduling, and simulation of random processes.

T. L. Saaty (Silver Spring, Md.)

627:
Gilbert, E. N. Optimal search strategies. J. Soc. Indust. Appl. Math. **7** (1959), 413–424.

From the author's summary: "In some search problems there is a choice of several different places in which to

search. To take a typical example imagine a man searching for a nut to fit an odd-sized bolt. Suppose that he has two bins full of assorted sized nuts mixed together. He can search only one bin at a time. However, if he is unsuccessful searching bin 1 he may eventually decide that bin 1 hasn't many nuts of the correct size and may switch his search to bin 2. If he still fails to find the nut in bin 2 he may eventually switch back to bin 1, etc. Assuming that he has fixed methods of searching places 1 and 2, his strategy is just the rule whereby he decides where to start searching and when to switch. In simple probabilistic models of this search problem we will find strategies which minimize the expected time spent searching." *J. Kiefer* (Ithaca, N.Y.)

628:

Haight, Frank A. Overflow at a traffic light. Biometrika **46** (1959), 420–424.

Formulas are derived for the transition probabilities from a queue of length x at the beginning of a green phase of a traffic light to a queue of length z at the end of the green phase. Traffic is assumed to have Poisson arrivals and regular departures whenever the queue is non-empty but new arrivals experience no delay if the queue is empty and the light is green.

G. Newell (Providence, R.I.)

629:

Zwinggi, Ernst. Methodische Bemerkungen zur Berechnung der Prämien, Deckungskapitalien und Gewinne in der Lebensversicherung. Mitt. Verein. Schweiz. Versich.-Math. **59** (1959), 251–260. (French, Italian and English summaries)

The author derives formulae for premiums, reserves and dividends of endowment assurances under the following conditions: The death benefit is evaluated by the continuous method while the premiums are evaluated according to their actual, discrete mode of payment. The paper is a slight condensation of an earlier article of the author [Giorn. Ist. Ital. Attuari **21** (1958), 1–12; MR **21** #7110]. *E. Lukacs* (Berkeley, Calif.)

630:

Vincent, Shirley E. An application of linear programming to agricultural economics. Appl. Statist. **9** (1960), 28–36.

631:

Wagner, Harvey M. On the distribution of solutions in linear programming problems. J. Amer. Statist. Assoc. **53** (1958), 161–163.

M. M. Babbar [J. Amer. Statist. Assoc. **50** (1955), 854–869; MR **17**, 380] investigated the distribution of the solutions of a linear programming problem if the data are stochastic, i.e., all the parameters of the problem are subject to probability distributions. The author criticises the article because the condition of feasibility (non-negativity of solutions) is neglected and the approach is not applicable in many concrete situations.

G. Tintner (Calcutta)

632:

Talacko, Joseph. Special methods of inventory by sampling if the population sets have approximately negative exponential distribution. Trabajos Estadíst. **10** (1959), 19–29. (Spanish summary)

633:

Zoroa, P. Saddle-matrix in rectangular games. Trabajos Estadíst. **10** (1959), 3–12. (Spanish. English summary)

A saddle matrix of a game matrix is a submatrix corresponding to sets of strategies (S_1, S_2, for the two players) such that while player 2 plays from S_2, any deviation from S_1 is dominated by a mixed strategy from S_1, and symmetrically. The author works out some exercises on saddle matrices. *J. Isbell* (Seattle, Wash.)

634:

Shapley, L. S. Equilibrium points in games with vector payoffs. Naval Res. Logist. Quart. **6** (1959), 57–61.

A "game with vector payoff" is represented by an m by n matrix (a_{ij}) of p-vectors a_{ij}. If the two players of the game employ respectively the mixed strategies $(x_1, \cdots, x_m)$, $(y_1, \cdots, y_n)$, the expected payment to the first player from the second is the p-vector $xAy = \sum_{i,j} x_i A_{ij} y_j$. The first player wants to increase the components of this vector and the second player to decrease them, but neither has assigned specific relative values to its components.

The p-vector a strongly dominates the p-vector b if $a_k > b_k$ $(k=1, \cdots, p)$; a weakly dominates b if $a \neq b$ and $a_k \geq b_k$ $(k=1, \cdots, p)$. The pair (x^*, y^*) of mixed strategies is a strong [weak] equilibrium point for the game if there is no mixed strategy x for which xAy^* weakly [strongly] dominates x^*Ay^* and no y for which x^*Ay^* weakly [strongly] dominates x^*Ay. Let α and β be non-negative p-vectors; let $[\alpha A, -\beta A] = \Gamma$ be the non-zero-sum two-person games whose payoffs to the players are respectively αa_{ij} and $-\beta a_{ij}$ when pure strategies i and j are used. Theorem: The weak [strong] equilibrium points of the vector-payoff game are precisely the equilibrium points of the game Γ for α and β which weakly [strongly] dominate the zero p-vector. *P. Wolfe* (Santa Monica, Calif.)

635:

Shapley, L. S. A solution containing an arbitrary closed component. Contributions to the theory of games, Vol. IV, pp. 87–93. Annals of Mathematics Studies, no. 40. Princeton University Press, Princeton, N.J., 1959. xi+453 pp. $6.00.

Let C be an arbitrary closed subset of the set of all non-negative $(n-3)$-tuples $(u_4, \cdots, u_n)$ for which $\sum_4^n u_j < 1$; and let J be the set of all n-tuples $(0, \frac{1}{2}u_0, \frac{1}{2}u_0, u_4, \cdots, u_n)$ for which $u_0 = 1 - \sum_4^n u_j$ and $(u_4, \cdots, u_n) \in C$. The set J is a subset of the "fundamental simplex" of imputations of an n-person game. Another set, closed, connected, and disjoint from J, is constructed in such a way that the union of this set with J constitutes the von Neumann-Morgenstern solution of a certain n-person game: a simple game with the winning coalitions $N-\{1\}$, $N-\{2\}$, $N-\{3\}$, and N. As the author states, the arbitrariness of J illustrates the great variety of solutions of a game which the classical theory permits, and is a ready

source of examples for the testing of conjectures about properties of solutions. *P. Wolfe* (Santa Monica, Calif.)

636:

Gurk, H. M.; Isbell, J. R. Simple solutions. Contributions to the theory of games, Vol. IV, pp. 247–265. Annals of Mathematics Studies, no. 40. Princeton University Press, Princeton, N.J., 1959. xi+453 pp. $6.00.

A simple solution [called pseudo-main solution in Isbell, Proc. Amer. Math. Soc. **6** (1955), 346–348; MR **17**, 57] of any n-person game (normalized, $0-1$) is a solution in which the ith player ($i=1, \cdots, n$) gets $x_i \geq 0$ or 0 in each imputation of the solution. The authors first derive conditions for the existence of simple solutions in general multi-person games. Among other things they introduce the notion of atomic games, generalizing that of simple games, and show that every simple solution of an n-person game is also a solution of some maximal atomic game. The bulk of the paper deals, however, with simple games. So called extreme simple solutions are studied, their uniqueness investigated and criteria for determining such solutions for certain games are given. These results are applied to investigate the solutions of certain weighted majority games and projective plane games. {The authors wish to state the correction "In the list of six person games, in the thirteenth game, '234' should be '156'."}

A. Dvoretzky (Jerusalem)

BIOLOGY AND SOCIOLOGY

· See also A10.

637:

Pearce, S. Clifford; Holland, Derek A. Some applications of multivariate methods in botany. Appl. Statist. **9** (1960), 1–7.

638:

Woodger, J. H. Studies in the foundations of genetics. The axiomatic method. With special reference to geometry and physics. Proceedings of an International Symposium held at the Univ. of Calif., Berkeley, Dec. 26, 1957–Jan. 4, 1958 (edited by L. Henkin, P. Suppes and A. Tarski), pp. 408–428. Studies in Logic and the Foundations of Mathematics. North-Holland Publishing Co., Amsterdam, 1959. xi+488 pp. $12.00.

Using the notation of his book [*Biology and language*, Cambridge Univ. Press, Cambridge, 1952] the author sets up an axiomatic system describing random mating in genetics and emphasizes the particular assumptions which are required but not always made explicit in the derivation of Mendel's First Law for the inheritance of a single character. *P. A. P. Moran* (Oxford)

INFORMATION AND COMMUNICATION THEORY

639:

Münzner, Hans. Über einige Grundbegriffe der Informationstheorie. Bl. Deutsch. Ges. Versicherungsmath. **4** (1959/60), 249–258. (English summary)

An elementary mathematical exposition of some basic ideas in the theory of information. In particular, the author considers the information content of a message or a symbol from a source, entropy or the average information content (per symbol) of a source, the redundancy of a "language," the channel and channel capacity.

S. Kullback (Washington, D.C.)

640:

Schmetterer, Leopold. Literaturbericht zur Informationstheorie. Bl. Deutsch. Ges. Versicherungsmath. **4** (1959/60), 259–266.

An exposition summarizing recent results and work in information theory. There is a bibliography of 76 items; nine of these items are from the period 1924–1950, the remainder from the period 1951–1959 with the years 1956 and 1957 most heavily represented.

S. Kullback (Washington, D.C.)

641:

Watanabe, Satosi. Information theoretical analysis of multivariate correlation. IBM J. Res. Develop. **4** (1960), 66–82.

Author's summary: "A set λ of stochastic variables, $y_1, y_2, \cdots, y_n$, is grouped into subsets, $\mu_1, \mu_2, \cdots, \mu_k$. The correlation existing in λ with respect to the μ's is adequately expressed by $C=\sum_{i=1}^{k} S(\mu_i)-S(\lambda) \geq 0$, where $S(\nu)$ is the entropy function defined with reference to the variables y in subset ν. For a given λ, C becomes maximum when each μ_i consists of only one variable, ($n=k$). The value C is then called *the total correlation in* λ, $C_{\text{tot}}(\lambda)$. The present paper gives various theorems, according to which $C_{\text{tot}}(\lambda)$ can be decomposed in terms of the partial correlations existing in subsets of λ, and of quantities derivable therefrom. The information-theoretical meaning of each decomposition is carefully explained. As illustrations, two problems are discussed at the end of the paper: (1) redundancy in geometrical figures in pattern recognition, and (2) randomization effect of shuffling cards marked 'zero' or 'one'."

S. Kullback (Washington, D.C.)

642:

Li, Hen Von. Generalization of the Shinbrot method for determining the optimal weight function for nonstationary random inputs. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Energet. Avtomat. **1959**, no. 3, 63–75. (Russian)

A signal $s(\cdot)$ is defined by $s(t)=g(t)+m(t)$ where $g(\cdot)$ is a polynomial and $m(\cdot)$ is a random process. Let $n(\cdot)$ be a noise uncorrelated with $m(\cdot)$, and H, K linear operators, with H given and K to be chosen so as to depend on $s(\cdot)+n(\cdot)$ in (O, T) only. With an error $e(\cdot)$ defined as $e(\cdot)=Hs(\cdot)-Ks(\cdot)-Kn(\cdot)$, the problem is to choose K so as to make $E\{e\}=0$ and $E\{e^2\}=\min$. This is solved by the now classical integral equation method [cf. Shinbrot, NACA Tech. Note No. **3791** (1956); J. Math. and Phys. **36** (1957), 121–129; MR **19**, 71, 1206]. The bulk of the paper is devoted to actual solutions of the equation in cases where the kernels can be approximated by functions that are polynomials in one variable; a simple explicit example is worked out in detail.

V. E. Beneš (Murray Hill, N.J.)

643:

Li, Hen Von. Generalization of the Shinbrot method for determining the optimal weight function for non-stationary random inputs in the case of white noise. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Energet. Avtomat. **1959**, no. 4, 177–187. (Russian)

Details of a special case (white noise) of the problem treated in the paper review above are given.

V. E. Beneš (Murray Hill, N.J.)

644:

Lomnicki, Z. A.; Zaremba, S. K. The asymptotic distributions of estimators of the amount of transmitted information. Information and Control **2** (1959), 260–284.

The authors consider a channel with input alphabet 1 to r and output alphabet 1 to s. Let p_{ij} denote the joint probability that the input is i and the output is j. The amount of information transmitted per symbol is defined to be

$$T = \sum_{ij} p_{ij} \log\{p_{ij}/p_{i\cdot}p_{\cdot j}\},$$

where $p_{i\cdot} = \sum_j p_{ij}$ and $p_{\cdot j} = \sum_i p_{ij}$. Assuming that successive channel inputs, and therefore outputs, are independent the authors determine the asymptotic distribution of various estimates $\hat{T}$ of T under a number of assumptions on p_{ij}. The authors also study the properties of systems which are separable in the sense that for every i and j, either $p_{ij}=0$ or $p_{ij}=\alpha_i p_{i\cdot}p_{\cdot j}$, where α_i is a postive constant.

E. Parzen (Stanford, Calif.)

645:

Judd, B. R.; Sutherland, N. S. The information content of nonsequential messages. Information and Control **2** (1959), 315–332.

A set of s distinct 'letters' from an 'alphabet' of size t are transmitted, the order being irrelevant, and the receiver prints r letters from the same alphabet. Under various assumptions the amount of information transmitted is calculated. The intended application is to psychological experiments on recognition and recall. Different performances cannot be readily compared when r, s, and t are not fixed, unless a measure of the amount of information is available. Experimental checks of the assumptions may be provided by future work.

I. J. Good (Teddington)

646:

Blum, Marvin; Gottschalk, Hirsch. On the bias errors of least squares digital filters. J. Soc. Indust. Appl. Math. **7** (1959), 205–214.

Let the input to a filter at time t ($t=0, 1, 2, \cdots$) be a sequence $Y_0, Y_1, \cdots, Y_t$, with $Y_t = \sum_{i=0}^{n} a_i t^i + N(t)$, where the a_i are unknown coefficients and $\{N(t)\}$ is a white noise process with zero mean. Let $Z_t^{(n)}$, $Z_t^{(n)} = \sum_{\tau=0}^{t} W_{t-\tau} Y_\tau$, be a linear, unbiased, minimum variance estimate of $P(t) = a_0 + a_1 t + \cdots + a_n t^n$. If the degree of $P(t)$ is greater than n, $Z_t^{(n)}$ will not be an unbiased estimate of $P(t)$. By employing an orthogonal polynomial representation for $P(t)$ the authors compute the bias error for the case where the degree of $P(t)$ is n', $n' > n$.

L. A. Zadeh (Berkeley, Calif.)

647:

Joshi, Devi Datt. L'information en statistique mathématique et dans la théorie des communications. Publ. Inst. Statist. Univ. Paris **8** (1959), 81–159.

This work consists of an introduction and four chapters: Chapter I, The fundamental theorem of information theory, discrete case; Chapter II, The problem of binary coding; Chapter III, The fundamental theorem, general case; Chapter IV, Information in mathematical statistics. The first chapter serves as an introduction to Shannon's theory [Shannon, Bell System Tech. J. **27** (1948), 379–423, 623–656; MR **10**, 133]. The author shows that entropy and capacity, the two basic concepts of the theory, are related to two different coding problems. The first, a translation problem, consists of finding an economical one-to-one correspondence between messages in an alphabet A of m letters and an alphabet B of s letters. The second, related to the capacity of a channel, consists of finding a procedure whereby an output message can be related to the input message with minimum error of identification. The author proves the fundamental theorem for the capacity of a binary symmetric channel. Because of the practical importance of channels transmitting messages which are sequences of two symbols only, the author devotes the second chapter to a study of binary coding. The message space consists of all sequences of length n made up of two symbols 0 and 1. The author considers the message space as an abelian group of order 2^n and type $(1, 1, \cdots, 1)$. The author defines distance in the message space as the sum of the absolute values of the differences between corresponding elements of two message vectors. He then considers subsets such that the distance between any two elements of the subset is greater than or equal to a given number. The author shows that there always exists an optimal code which is a subgroup, and also derives two inequalities for the number of elements of the largest optimal code. In the third chapter the author generalizes the Shannon theory to abstract spaces. A channel is defined as a measurable space $(Y, \mathscr{Y})$ on which is defined a probability measure ν_x for every point x belonging to a measurable space $(X, \mathscr{X})$. The capacity of a channel is defined as the supremum of the rate of transmission relative to a family of probability measures of possible input messages. For the abstract formulation, the author derives a theorem which is an analogue of Shannon's fundamental theorem, following the procedure of Feinstein [Research Laboratory of Electronics, M.I.T., Tech. Rep. No. **282** (1954); Trans. I.R.E. **PGIT-4** (1954), 2–22; MR **17**, 1098; **19**, 516]. The author remarks that it is not possible to define entropy in the general case and therefore defines the capacity of a channel, as above, without introducing entropies as is the usual case. It should be noted that for the general case, a generalized entropy has been defined, and the rate of transmission defined as usual in terms of the generalized entropy, as well as a version equivalent to that given in this paper, by Pérez [Transactions of the first Prague conference on information theory, statistical decision functions, random processes held at Liblice near Prague from November 28 to 30, 1956, pp. 183–208; 209–243. Publishing House of the Czechoslovak Academy of Sciences, Prague, 1957; MR **20** #6325, #6326]. In the fourth chapter the author considers applications of the concept of information in mathematical statistics. The author considers information as a positive numerical function defined over a measurable space of observations, additive for independent observations, and invariant under transformations which are sufficient statistics. The author derives certain results relating information, hypothesis testing, and statistical

discrimination. Space does not permit giving details here. *S. Kullback* (Washington, D.C.)

648:

Loeb, Julien. Une interprétation physique de l'ambiguïté de Shannon. Ann. Télécommun. **13** (1958), 78–82.

Résumé de l'auteur: "Un élément de circuit bruyant peut généralement être représenté par une série de probabilités de transition, c'est-à-dire des probabilités a_{kl} pour que chaque symbole S_k transmis soit (faussement) reçu comme le symbole S_l. On construit une matrice avec ces probabilités de telle façon que les propriétés statistiques des éléments de circuit en cascade puissent être calculées au moyen de la multiplication des matrices. Lorsqu'il y a de nombreux symboles et que le bruit est additif (modulation d'amplitude et types semblables de modulation) les matrices deviennent des grilles et le résultat est beaucoup plus simple car ces grilles peuvent être multipliées de la même façon que le sont les simples polynômes algébriques. On trouve, dans ce cas, l'interprétation physique de l'ambiguïté de Shannon. L'instrument mathématique qui a été développé ici peut être appliqué à des problèmes plus compliqués, tels que des éléments de circuit non linéaires, des bruits non gaussiens ou des codeurs." *S. Kullback* (Washington, D.C.)

649:

Rozenberg, L. A.; Skorneva, M. I. On filtering of sequences of random variables. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Energet. Avtomat. **1959**, no. 3, 55–62. (Russian)

Let $\{\eta_n\}$ and $\{\zeta_n\}$ be uncorrelated, stationary, discrete time parameter random processes with known correlation functions. Let $\xi_n = \eta_n + \zeta_n$. The authors consider the problem of finding minimum variance estimates of η_n in the form $\sum_{j=0}^{m} \alpha_j \xi_{n-j}$ for the case where the α_j are functions of l parameters $q_1, \cdots, q_l$, with $l < m+1$. Explicit results are found for the special case where $l=1$ and $\alpha_k = q_1(1-q_1)^k$. Another case considered is that where η_n is a polynomial in n with unknown coefficients. In this case and, in particular, for large m, the authors' method is considerably less effective than the methods based on the spectrum shaping technique in which the number of linear equations to be solved for the α_j does not depend on m.

L. A. Zadeh (Berkeley, Calif.)

650:

Lee, Y. W. Statistical filtering and prediction. Nuovo Cimento (10) **13** (1959), supplemento, 430–454.

Expository paper. [See, for example, Wiener, *Extrapolation, interpolation and smoothing of stationary time series*, Wiley, New York, 1950; MR **11**, 118 and Bendat, *Principles and applications of random noise theory*, J. Wiley, 1958; MR **21** #4489; Chapter 4.]

K. S. Miller (New York)

651:

Slepian, D. Coding theory. Nuovo Cimento (10) **13** (1959), supplemento, 373–388.

This is a talk given in a course on information theory. As such it is a good, careful introduction to the topic of coding theory and does not attempt to be a critical survey of the current state of the field.

R. W. Hamming (Murray Hill, N.J.)

652:

Hocquenghem, A. Codes correcteurs d'erreurs. Chiffres **2** (1959), 147–156. (English, German and Russian summaries)

"The paper is a generalization of Hamming's work. The author gives a coding system available to correct k errors in a transmission of binary digits."

The method used involves introducing a ring and examining congruences in it. The method appears to be semi-experimental in part, thus satisfying practical needs while leaving some theoretical aspects still open.

R. W. Hamming (Murray Hill, N.J.)

653:

Korolev, L. N. Coding and compression of codes. Dokl. Akad. Nauk SSSR (N.S.) **113** (1957), 746–747. (Russian)

Formulations are given of certain results relating to the problem of the practical discovery of methods of optimal coding of words from a finite set of words—a "dictionary".

Theorem 1. There exists a number $d(r, N)$, depending only on the number of letters r of the coding alphabet and the number of words N of the dictionary being coded, such that for any one-one coding the sum of the lengths of all encoded words will be greater than or equal to d.

An operation ∇_n is defined of "contracting codes to length n", associating to the code of a word of arbitrary length the code of a word of length n in the same coding alphabet.

Theorem 2. There exists a class of contracting operators ∇_n for which the probability that the application of ∇_n to all codes of the dictionary violates the one-one correspondence of the coding approaches zero with increasing n.

Reviewer's remark. The formulation of theorem 2 is somewhat indefinite; an example of the author illustrates the application of this theorem: if a dictionary is regarded as a random sample of all encoded words in the r-letter coding alphabet, there exists an operation ∇_n such that its application to a randomly chosen code gives any word of length n with equal probability $1/r^n$.

A. P. Eršov (RŽMat **1958** #4026)

SERVOMECHANISMS AND CONTROL

See also 335, 346.

654:

Woodrow, R. A. Data fitting with linear transfer functions. J. Electronics Control (1) **6** (1959), 454–480.

An unsophisticated discussion of simple criteria leads the author to the conclusion that the mean square error criterion is, both mathematically and physically speaking, the best. Only time invariant, infinite settling time systems are considered. {There are some mathematical errors.} *K. S. Miller* (New York)

655:

Bridgland, T. F., Jr. A linear algebraic formulation of the theory of sampled-data control. J. Soc. Indust. Appl. Math. **7** (1959), 431–446.

The purpose of this paper is to give a formulation of sampled-data control theory in purely algebraic terms,

avoiding complications that arise from emphasizing analogies with the analytical theory of continuous-data control. Let σ denote the linear vector space of sequences $x=(x_0, x_1, x_2, \cdots)$ of complex numbers. By a linear filter is meant a transformation $\sigma \to \sigma$ of the form

$$y_n = \sum_{k=0}^{\infty} a_{nk}x_k \quad (n = 0, 1, 2, \cdots),$$

where the a_{nk} are complex numbers such that $a_{nk}=0$ for $k>n$. A linear filter is called constant if $a_{nk}=a_{n-k}$ for all n, k. The linear theory of sampled-data control is defined broadly as the theory of such constant and non-constant linear filters. Besides some general discussion of the new concept of sampled-data control, the paper gives a summary, mostly without proofs, of the important properties of σ and of the infinite matrices (a_{nk}) defining linear filters. A major objective of the paper is an interpretation, in algebraic terms, of the widely used z-transform. This objective is achieved by interpreting the symbol z^{-1} of z-transform theory not as a complex variable, but as a displacement operator E^{-1}, applicable to sequences and defined by the formula $E^{-1}(x_0, x_1, x_2, \cdots) = (0, x_0, x_1, \cdots)$. However, in the consideration of questions of stability the author is led to revert to something like the usual z-transform theory, with its complex variables and analytic functions. Although this turn of the theory is contrary to the author's basic intentions, it seems likely that something of the kind is necessary if the theory is fully to satisfy practical requirements.

L. A. MacColl (New York)

656:

Bellman, Richard; Holland, John; Kalaba, Robert. On an application of dynamic programming to the synthesis of logical systems. J. Assoc. Comput. Mach. **6** (1959), 486–493.

Consider a sequential machine in which the state vectors are of the form $x=(x_1, x_2, \cdots, x_m)$, with $x_i=0, 1$ $(i=1, 2, \cdots, m)$, $\|x-y\| = \sum_{i=1}^{m} |x_i-y_i|$. Let $x^{(t)}$, $t=0, 1, 2, \cdots$, denote the state at the time t, and let q_t be the input at time t. By using the techniques of dynamic programming the authors solve the following problem. Given (i) an initial state $x^{(0)}=\alpha$; (ii) a terminal state β; (iii) the state transition relation $x^{(t+1)}=f(x^{(t)}, q_t)$ which characterizes the machine, find a sequence of inputs $q_0, q_1, \cdots, q_{N-1}$ which minimizes the distance $\|x^{(N)}-\beta\|$ for fixed N. Once this problem is solved, it is a simple matter to find an input of minimal length which takes the system from α to β. In the latter case, the solution yielded by dynamic programming is identical with that yielded by considering the set of states S from which β can be reached in one step, then considering all states from which S can be reached in one step, and so on.

L. A. Zadeh (Berkeley, Calif.)

657a:

Sawaragi, Yoshikazu; Sugai, Narinobu. On the stability problems of non-linear control system with a stationary random noise. Tech. Rep. Engrg. Res. Inst. Kyoto Univ. **9** (1959), 129–140.

657b:

Sawaragi, Yoshikazu; Sugai, Narinobu. On the "signal stabilization of a non-linear control system" by impressing a sinusoidal signal. Tech. Rep. Engrg. Res. Inst. Kyoto Univ. **9** (1959), 141–157.

An investigation of the existence of sustained oscillations in nonlinear circuits in which an external signal (sinusoidal or gaussian) is injected at some point in a loop comprising a memoryless nonlinear two-pole and a linear time-invariant two-pole. The authors assume that a primary effect of the external signal consists in changing the "equivalent gain" of the nonlinear device for limit cycle oscillations. Neither this assumption nor other assumptions concerning the independence and normality of signal amplitudes are adequately justified.

L. A. Zadeh (Berkeley, Calif.)

658:

Aramă, O. Observations sur les schémas électriques à résistances et contacts. Acad. R. P. Romîne. Fil. Cluj. Stud. Cerc. Şti. Ser. I. **6** (1955), nos. 3–4, 75–85. (Romanian. Russian and French summaries)

Relay-contact schemes with connections in series and in parallel are replaced by others with multipoles, and such schemes are put into a disjunctive form with respect to the intermediate and executive elements. See Lunc [Izv. Akad. Nauk SSSR. Ser. Mat. **16** (1952), 405–436; MR **14**, 606] and Moisil [Acad. R. P. Romîne. Stud. Cerc. Mat. **6** (1955), 7–53; MR **17**, 328].

F. Haimo (St. Louis, Mo.)

AUTHOR INDEX

PART B

Abate, E. ... 556
Adams, Ernest W. ... 338
Adkins, J. E. ... 400
Adlam, J. H. ... 490
Aeschlimann, Florence ... 519
Aleksandryan, R. A. ... 367
Alekseev, A. I. ... 583
Allen, J. E. ... 490
Amati, D. ... 569
Amstutz, Pierre ... 510
Aramă, O. ... 658
Arutyunyan, N. H. ... 367
Ashkin, J. ... 572
Azorín Poch, Francisco. 306, 624
Babičev, A. I. ... 435
Bach, Günter ... 259
Bailey, Norman T. J. ... 299
Baker, W. E. ... 404
Baltrukonis, J. H. ... 377
Bartlett, M. S. ... 308
Bass, F. G. ... 502
Bearman, Richard J. ... 352
Bellman, Richard ... 656
Benedetti, Carlo ... 262
Bertram, G. ... 323
Besshо, Masatoshi ... 428
Bhattacharya, Nikhilesh ... 264
Bingel, Werner A. ... 567
Biot, A. ... 473, 474, 475
Bkouche, Rudolphe ... 611
Blalock, H. M., Jr. ... 275
Blatt, J. M. ... 351
Bleich, H. H. ... 340
Bloh, V. I. ... 368
Blum, Marvin ... 646
Bodine, R. Y. ... 388
Boerboom, A. J. H. ... 480, 497
Boffi, Vinicio C. ... 356
Boley, B. A. ... 361
Borgnis, F. ... 492
Borodačev, N. M. ... 398
Braunbek, Werner ... 506
Bressan, Aldo ... 341
Bridgland, T. F., Jr. ... 655
Brinis, Elisa ... 395
Bryak, H. ... 505
Budiansky, Bernard ... 434
Builder, Geoffrey ... 597
Burley, D. M. ... 348
Burmistrov, E. F. ... 385
Bush, William B. ... 472
Caianiello, E. R. ... 550
Cairo, Lorenzo ... 509
Calleja Siero, Anselmo ... 277
Case, K. M. ... 357
Chakravorti, Amitava ... 371
Chang, C. C. ... 450
Chang, K. T. ... 324
Chapman, Douglas G. ... 284
Chatelet, F. ... 280
Cheney, E. W. ... 316
Chirol, Henri ... 330
Čičerin, A. G. ... 532
Clenshaw, C. W. ... 315
Cohen, A. Clifford, Jr. ... 276
Cohen, Arthur ... 292
Coleman, Bernard D. ... 413
Collar, A. R. ... 455
Collatz, Lothar ... 322
Cooper, D. C. ... 515
Costa de Beauregard, Olivier. 599
Cowden, Dudley J. ... 286
Cox, D. R. ... 303
Cramér, Harald ... 246
Crampin, Joan ... 598
Crandall, Stephen H. ... 511
Cruon, R. ... 254
Csonka, P. ... 381
Cyrlin, L. E. ... 498
Datzeff, Assène ... 522, 523
Dautcourt, Georg. ... 610
DeGroot, M. H. ... 281
Deming, Lola S. ... 260, 261
Dem'yanov, Yu. A. ... 443
Deo, B. ... 537
Desloge, Edward A. ... 350
Destouches, Jean-Louis ... 520
DiMaggio, F. L. ... 340
Di Prima, R. C. ... 369
Diukalov, A. N. ... 457
Drake, D. G. ... 426
Dreicer, H. ... 483
Druyanov, B. A. ... 402
Dunn, Olive Jean ... 283
Durand, Philippe ... 495
Dwass, Meyer ... 272
Eganyan, V. V. ... 362
Ehrmann, Hans ... 311
Eisenschitz, R. ... 353
Ellington, J. P. ... 387
Elrod, H. G. ... 438
Englert, F. ... 524
Estrin, M. I. ... 380
Fabri, E. ... 556
Fadell, Edward ... 233
Faĭnberg, Ya. B. .. 482, 484, 493
Farber, Erich A. ... 596
Fedorov, I. V. ... 412
Ferguson, Harry ... 427
Ferraro, V. ... 485
Février, Paulette ... 518
Fischer, H. R. ... 225
Fischer, J. ... 494
Fisz, M. ... 273
Fletcher, H. J. ... 374
Floyd, Robert W. ... 313
Foote, Richard J. ... 287
Ford, F. A. J. ... 256
Fourès-Bruhat, Yvonne ... 606
Fradlin, B. N. ... 382
Frantz, Lee M. ... 540
French, J. B. ... 560
Frisch, H. L. ... 359
Fujii, Tetsu ... 514
Fujita, S. ... 584
Fukuda, Nobuyuki ... 593
Gabarro, Leandro ... 509
Gabriel, K. R. ... 249
Gardner, J. W. ... 538
Geffroy, Jean ... 245
Ghinea, Monique ... 320
Gilbert, E. N. ... 257, 627
Ginsburg, Th. ... 449
Glass, Suzanne ... 295
Gnédenko, B. V. ... 240
Göcke, Hermann ... 344
Goldberg, J. N. ... 600
Goldberg, M. A. ... 366
Goldstein, A. A. ... 316
Gorbatenko, M. F. ... 484
Gottfried, K. ... 533
Gottschalk, Hirsch ... 646
Govorun, N. N. ... 508
Graffi, Dario ... 436
Granovskiĭ, Ya. I. ... 570
Grawert, Gerald ... 544
Green, T. S. ... 486
Grenander, Ulf. ... 247
Gribov, V. N. ... 552
Grigoryan, S. S. ... 456
Grodzovskiĭ, G. L. ... 451, 457
Grozdovskii, G. L. = Grodzovskiĭ, G. L.
Gruber, Boris ... 418
Guderley, Karl Gottfried ... 427
Gupta, K. K. ... 590
Gurk, H. M. ... 636
Guttman, Irwin ... 285
Haag, R. ... 531
Hagedorn, R. ... 528
Haight, Frank A. ... 241, 628
Hall, G. G. ... 558
Hallert, B. ... 620
Hamada, Minoru ... 393
Hara, O. ... 554
Hattori, Tsuyoshi ... 464
Haus, H. A. ... 512
Haynsworth, Emilie V. ... 318
Helfand, E. ... 359
Henderson, Roland G. ... 615
Hennequin, Paul-Louis ... 250
Hermes, Hans ... 337
Hilgevoord, J. ... 530
Hiroike, Kazuo ... 582
Hocquenghem, A. ... 652
Hoff, N. J. ... 408
Holland, Derek A. ... 637
Holland, John ... 656
Hoquetis, Bernard P. ... 511
Hosford, John E. ... 305
Hoskin, N. E. ... 515
Hudimoto, Hirosi ... 293
Hwang, Chintsun ... 410
Ichikawa, Yoshi H. ... 534
Il'yušin, A. A. ... 407
Imamura, Tsutomu ... 553
Ioffe, I. A. ... 517
Iordanskiĭ, S. V. ... 429
Isbell, J. R. ... 636
Jahsman, W. E. ... 408
Jarmain, W. R. ... 564
Johnson, N. L. ... 243, 265
Jones, R. Morley ... 301
Joshi, Devi Datt ... 647
Jovi, Maria Grazia ... 507
Judd, B. R. ... 645
Kalaba, Robert ... 656
Kaliski, Sylwester ... 405, 406
Kaner, E. A. ... 502
Kappler, M. O. ... 334
Kapur, M. N. ... 302
Katzin, Martin ... 503
Kaufmann, A. ... 254
Kawaguti, Mitutosi ... 439, 440
Kawata, T. ... 251
Keshava Rao, M. N. ... 373
Keston, Harry ... 253
Khaskind, M. D. ... 452
Khatri, C. G. ... 267
Khintchine, A. Ia. ... 240
Khomenko, V. S. ... 452
Kimura, Katsumi ... 422
Kimura, Masao ... 422
Kiselev, M. I. ... 471
Kito, Fumiki ... 389, 390
Klotz, A. H. ... 607
Kobayakawa, Keizo ... 553
Kočina, N. N. ... 458
Koide, Shoichiro ... 568
Kondurar', V. T. ... 347
Konisi, Gaku ... 531
Koo, Benjamin Y.-C. ... 503
Korolev, L. N. ... 653
Körper, Klaus ... 487, 488, 489
Kosmodamianskiĭ, A. S. ... 363
Kostyukov, A. A. ... 431
Kovrižnyh, L. M. ... 466
Kramer, Clyde Young ... 295
Krishna Iyer, P. V. ... 274
Kröner, E. ... 585
Kronsbein, John ... 596
Kučerov, R. Ya. ... 355, 513
Kupperman, Morton ... 279
Kurdin, N. S. ... 375
Lansraux, G. ... 280
Lauwerier, H. A. ... 430
Lebedev, V. L. ... 258
Lebowitz, J. L. ... 359
Ledersteger, K. ... 623
Lee, T. D. ... 580, 581
Lee, Y. W. ... 650
Lévy, M. ... 578
Li, Ching-hai ... 622
Li, Hen Von ... 642, 643
Liénard, Pierre ... 461
Limber, D. Nelson ... 613
Lin, Chun C. ... 565
Ling, Chih-Bing ... 365
Liu, Hui-lin ... 621
Loeb, A. L. ... 419, 420
Loeb, Julien ... 327, 648
Löfgren, Tor. ... 421
Loinger, A. ... 527
Lomnicki, Z. A. ... 310, 644
Longuet-Higgins, M. S. ... 478
Loskutov, Yu. M. ... 573
Lowry, Edward S. ... 601
Łukaszuk, L. ... 574
Lundgren, T. S. ... 450
Lüst, R. ... 467
Luzzatto, G. ... 551
Lyubimov, G. A. ... 470
Machnig, Oskar ... 394
Magiros, Demetrios G. ... 346
Maĭkapar, G. I. ... 453
Malécot, G. ... 252
Mandel, John ... 304
Mandelbrot, Benoit ... 625
Mandelstam, Stanley ... 575, 576
Manukyan, M. M. ... 367
Marathay, A. S. ... 477
Martens, Henrik H. ... 335
Martin, J. L. ... 525, 526
Martin, Paul C. ... 588
Masterov, E. P. ... 463
Mathur, V. S. ... 590
Matthyssе, Steven W. ... 350
McCallion, H. ... 387
McCrea, W. H. ... 598
McCune, J. E. ... 469
McNally, D. ... 598
Meiron, J. ... 479
Mével, Jean ... 504
Million, B. ... 280
Mills, Robert L. ... 540
Mishra, R. S. ... 609
Misra, S. P. ... 604
Moore, R. G., Jr. ... 562
Mori, Hazime ... 354
Morris, I. L. ... 420
Moskalenko, V. A. ... 592
Mossakovskiĭ, V. I. ... 432
Mróz, Zenon ... 409
Mühlschlegel, Bernhard ... 589
Munschy, Gilbert ... 586
Münzner, Hans ... 639
Murakami, Terutosi ... 349
Nachbar, W. ... 408
Nadler, Jack ... 281
Naylor, D. ... 448
Newman, E. ... 600
Newton, T. D. ... 563
Niblett, G. B. F. ... 486
Niu, Keishiro ... 424
Noll, Walter ... 413
Novackiĭ, Votol'd = Nowacki, Witold
Novozhilov, Yu. ... 529
Nowacki, Witold ... 414, 415
Nožička, František ... 336
Nozières, Philippe ... 417
Öba, R. ... 425
Ogawa, Junjiro ... 298
Ogimoto, Takesi ... 531
Okamoto, Masashi ... 244
Okun, L. B. ... 549
O'Raifeartaigh, L. ... 577
Osiecki, Jan ... 405, 406
Ottoson, L. ... 620
Overend, John ... 566
Ozaki, Shoji ... 546
Pachares, J. ... 263
Papapetrou, Achilles ... 602
Pearce, S. Clifford ... 637
Pearson, E. S. ... 266
Peres, A. ... 605
Petzold, Joachim ... 544
Pham, Daniel ... 320

Philip, John R. 446
Phillips, Charles A. 333
Pillai, K. C. S. 268, 269
Pinte, R. 614
Pluvinage, Philippe 586
Poincelot, Paul 491
Pollaczek, Félix 255
Pollak, H. O. 257
Polyakova, A. L. 462
Popov, G. Ya. 376
Proriol, Joseph 586
Rabinovič, B. I. 386
Redmond, Peter J. 547
Reid, W. H. 441
Reiss, Howard 359
Reissner, E. 378
Remizova, N. I. 379
Rennie, B. C. 411
Rikenglaz, L. É. 355, 513
Rikitake, Tsuneji 618, 619
Robinson, Allan R. 423
Rockmore, R. M. 561
Rodberg, Leonard S. 541, 542
Rogers, Gerald S. 290
Romanovskiĭ, Yu. M. 392
Rosen, N. 605
Rosenblatt, S. 437
Rosendahl, Gottfried R. 476
Rozenberg, L. A. 649
Ruben, Harold 242
Rudik, A. P. 549
Rudis, M. A. 383
Rues, Dietrich 459
Rutkovič, B. N. 493
Rutishauser, Heinz 317
Rvačev, V. L. 432
Ryazanov, E. V. 468
Sadowsky, Michael 366
Sah, Priyamvada 353
Sahnovskiĭ, S. M. 382
Salamatov, Dł. 433
Šapiro, I. S. 579
Sauer, R. 325
Savage, I. Richard 289
Sawada, K. 561
Sawaragi, Yoshikazu. 657a, 657b
Scarf, F. L. 548
Scherer, J. R. 566
Scheube, Hans-Georg 616
Schmetterer, Leopold 640
Schmidt, R. 384
Schmutzer, Ernst 603
Schröder, Johann 319
Schwinger, Julian 588
Sciama, Antoine 332
Sears, W. R. 469
Segawa, Wataru 401
Sen, Pranab Kumar 288
Sen, S. N. 460
Sevruk, I. G. 516
Shah, B. V. 296
Shapley, L. S. 634, 635
Shen, Mok-Kong 360
Siebert, Manfred 617
Siebert, William H. 511
Simasaki, Tamedi 424
Simon, R. 614
Singh, Daljit 300
Sinha, K. P. 568
Sitenko, A. G. 539
Skalak, Richard 343
Skopec, M. B. 444
Skornova, M. I. 649
Slepian, D. 651
Smythe, W. R. 496
Sokolov, A. A. 573
Soo, Shao L. 445
Spencer, T. W. 399
Sredniawa, B. 577
Steel, Robert G. D. 282
Steginsky, Bernard 465
Stein, Charles 278
Stepanov, R. D. 372
Stoller, David S. 314
Strauss, Martin 594
Strelkov, S. P. 392
Sugai, Narinobu 657a, 657b
Sutherland, N. S. 645
Sveklo, V. A. 396
Swihart, James C. 591
Takács, L. 248
Talacko, Joseph 632
Talmi, Igal 559
Tanabe, Yukito 568
Tani, Smio 543
Taylor, J. C. 555
Terreaux, Ch. 577
Thirring, W. 571
Thiry, Y. 339
Tienzo, Benjamin P. 268
Tihonov, A. N. 500
Tikkiwal, B. D. 307
Tokarev, V. V. 457
Tolstyh, A. I. 457
Tong, Kin N. 345
Törnebohm, Håkan 595
Treder, Hansjürgen 602
Trefftz, E. 557
Tukey, John W. 309
Urban, A. 608
Uretsky, Jack L. 547
Uzdalev, A. I. 370
Vartak, Manohar Narhar ... 297
Vincent, Shirley E. 630
Vitale, B. 569
Vogel, Théodore 342
Volinez, G. 479
Vorob'ev, N. F. 454
Vorob'ev, Yu. V. 481
Vorovič, I. I. 363
Wada, Yasushi 593
Wagner, Harvey M. 631
Wait, James R. 501
Walsh, John E. 291, 294
Wannier, Gregory H. 416
Warmus, M. 328
Wasserman, Robert H. 447
Watanabe, Satosi 641
Watanabe, Yoshio 364
Weibull, Christer 270
Weiler, H. 271
Weinberger, H. F. 326
Weinmann, A. 536
Weinstein, M. A. 499
Wempner, G. A. 384
Wess, J. 587
White, J. E. 397
Wilkinson, J. H. 321
Wilson, Raymond H., Jr. 329
Wimmel, Hermann K. 358
Winter, Jacques 442
Woodger, J. H. 638
Woodrow, R. A. 654
Wynn, P. 312
Yamamoto, Hiroshi 612
Yamamoto, Masaaki 534
Yang, C. N. 580, 581
Yarymovych, M. I. 343
Yêm, Pham Xuân 521
Yu, Yi-Yuan 391
Zadoyan, M. A. 403
Zaidi, M. H. 535
Zaremba, S. K. 310, 644
Zinnes, I. I. 545
Zorca, P. 633
Zwinggi, Ernst 629

Centre National d'Études des Télécommunications 331
Operations Research Center, M.I.T. 626

291, 294
... 416
... 328
... 447
... 641
... 364
... 270
... 271
... 326
... 536
... 490
... 384
... 587
... 397
... 321
.. 329
... 358
... 442
... 638
... 654
... 312
... 612
... 534
580, 581
... 343
... 521
... 391
... 403
... 535
310, 644
... 545
... 633
... 629

des
... 331
ter,
... 626